Nonlinear Adaptive Robust Control of Single-Rod Electro-Hydraulic Actuator With Unknown Nonlinear Parameters

Cheng Guan and Shuangxia Pan

***Abstract*—In this paper, a nonlinear adaptive robust control method is presented for a single-rod electro-hydraulic actuator with unknown nonlinear parameters. Previous adaptive control methods of hydraulic systems always assume that the system's unknown parameters occur linearly, but, in a practical hydraulic system, unknown nonlinear parameters, which enter the system equations in a nonlinear way, are common; for example, when the original control volumes are unknown or change, uncertain nonlinear parameters will exist. The proposed control method in this paper is to present a nonlinear adaptive controller with adaptation laws to compensate for the uncertain nonlinear parameters due to the varieties of the original control volumes. The main feature of the scheme is that a novel-type Lyapunov function is developed to construct an asymptotically stable adaptive controller and adaptation laws. Furthermore, by combining backstepping techniques and a simple robust control method, the whole system's controller and adaptation laws are presented, which can compensate for all unknown parameters and uncertain nonlinearities. The experimental results show that the nonlinear control algorithm, together with the adaptation scheme, gives a good performance for the specified tracking task in the presence of unknown nonlinear parameters.**

***Index Terms*—Adaptive control, electro-hydraulic system, Lyapunov method, robust control, unknown nonlinear parameter.**

I. Introduction

HYDRAULIC systems have been widely used in industry by virtue of their small size-to-power ratios and the ability to produce very large force and torque. For example, they have been used in robot manipulators [1], hydraulic anti-lock braking systems [2], hydraulic elevators [3], and active suspension system, [4]. However, the dynamic behavior of hydraulic systems is highly nonlinear due to phenomena such as nonlinear servo valve flow-pressure characteristics and variations in control fluid volumes and associated stiffness, which, in turn, cause difficulties in the control of such systems. Aside from the nonlinear nature of hydraulic dynamics, hydraulic systems also have a large extent of model uncertainties due to parametric uncertainties and uncertain nonlinearities.

In the past, linear control theory has been used in much of the work on hydraulic control systems, such as in [5]–[8]. However, some important dynamic information may also be lost if the hydraulic servo system is linearized during the design. Thus, it is important to select a nonlinear control method that is particularly suitable for hydraulic servo systems.

First, state feedback-precise linearization techniques have been used in some research works [9]–[11]. However, this control method did not account for system uncertainties, so some works [2], [3], [12]–[14] adopted the sliding mode control method in electro-hydraulic systems based on linearization techniques. Since adaptive control is a valid method to solve system uncertainties, particularly uncertainties derived from uncertain parameters, many kinds of nonlinear adaptive control schemes have been adopted in hydraulic control systems to compensate for system uncertainties from uncertain parameters, such as sliding mode adaptive control [4], feedback-precise linearization adaptive control [15], and nonlinear adaptive control based on backstepping techniques [16]–[19]. For example, Alleyne and Liu [16], [17] applied the nonlinear adaptive control based on backstepping method to the force control of electro-hydraulic systems, and Yao *et al.*. [18], [19] applied nonlinear adaptive robust control (ARC) in double-rod and single-rod cylinder hydraulic systems based on backstepping techniques to compensate for the uncertainties from both parametric uncertainties and uncertain nonlinearities. Recently, some novel nonlinear adaptive control methods have been presented for hydraulic systems [20]–[23]. These nonlinear adaptive control laws not only solved the control problem coming from uncertain nonlinear systems successfully in some conditions, but also demonstrated that nonlinear control schemes can achieve a better performance than conventional linear controllers can.

However, one of the assumptions in these adaptive schemes is that the original total control volumes between the servo valve and cylinder, including volume of the servo valve, pipelines, and cylinder chambers, are certain and known. The previous assumption makes the system all unknown parameters occur linearly. However, in realistic hydraulic control systems, the original total control volumes are uncertain. The volume of the servo valve can be neglected since it is smaller correspondingly, but, in many cases, the volume of pipelines is too large to be neglected, and, moreover, it significantly affects the performance of the hydraulic control system. Nevertheless, the exact volume of pipelines is very difficult to obtain, and the length of pipelines may be variable in different work places to the same control system. Furthermore, the different original position of the cylinder piston can also make the original total control

Manuscript received January 3, 2007; revised May 4, 2007. Manuscript received in final form June 17, 2007. Recommended by Associate Editor Landers. This work was supported in part by the China Postdoctoral Science Foundation under Grant 20060401038.

The authors are with the Mechanical Design Institute, Zhejiang University, Hangzhou 310027, China (e-mail: gchzju@hotmail.com; psxx@zju.edu.cn).

Digital Object Identifier 10.1109/TCST.2007.908195

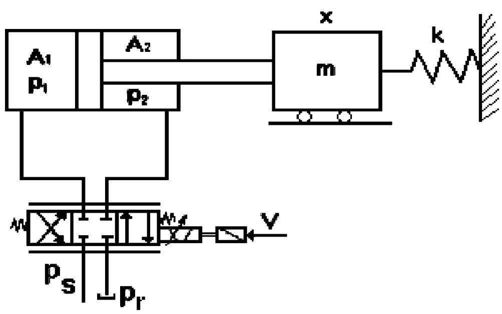

Fig. 1. Schematic diagram of the hydraulic system.

volumes change in different working processes. Therefore, the original total control volumes (mainly including pipelines and cylinder chamber) are uncertain but important, so, in practical hydraulic systems, unknown nonlinear parameters, which enter the system equations in a nonlinear way, are common, and previous adaptive schemes of hydraulic systems cannot solve the problem from the unknown nonlinear parameters. Hence, constant approximate estimated values of these volumes are always used, but this makes the performance of the control system unsatisfactory in some cases. Therefore, it is a challenging project to propose a nonlinear adaptive control method that can be used in electro-hydraulic system with unknown nonlinear parameters due to variations of the original total control volume.

In this paper, by utilizing a practice property of the hydraulic systems, we develop a special novel-type Lyapunov function to construct a Lyapunov-based controller and nonlinear parameter update laws and combine the backstepping method to provide the whole system controller and all unknown parameters update laws. Moreover, a simpler robust method is used to solve the problem derived from unmodeled uncertainties.

To test the proposed advanced nonlinear adaptive robust control strategy, an experiment is done. Experimental results have been obtained for the position tracking control of the hydraulic cylinder. Experimental results verify the higher performance nature of the proposed nonlinear adaptive robust control approach in comparison to the proposed control without adaptation laws.

This paper is organized as follows. In Section II, problem formulation and the detailed nonlinear model are presented. In Section III, the proposed nonlinear adaptive robust control with unknown nonlinear parameters strategy is given. In Section IV, the experiment is set up and the results are discussed. In Section V, the conclusions are presented.

II. Problem Formulation and Dynamic Models

The hydraulic system shown in Fig. 1 is comprised of a single-rod cylinder, a $4/3$-way servo valve, and a load force. The goal is to have the load track any specified motion trajectory as closely as possible. In the following, the nonlinear dynamical model will be given.

The dynamics of the force balance can be described by

$$P_1 A_1 - P_2 A_2 = m\ddot{x} + Kx + F_l(t) \tag{1}$$

where x is the displacement of the load, m is the mass of the load, P_1 and P_2 are the pressure inside the two chambers of the cylinder, respectively, A_1 and A_2 are the ram area of the two chambers, respectively, K is the effective bulk modulus of spring, and $F_l(t)$ is lumped uncertain nonlinearities due to external disturbances, the unmodeled friction forces, damping viscous friction forces on the load and the cylinder rod, and other hard-to-model terms.

Now, with the development of seal techniques, the external leakage flow is almost zero, so the external leakage of the cylinder is neglected here, and then the dynamics of cylinder oil flow can be written as [24]

$$\begin{cases} A_1\dot{x} + C_t\left(P_1 - P_2\right) + \frac{V_{01}+A_1 x}{\beta_e}\dot{P}_1 = Q_1 \\ A_2\dot{x} + C_t\left(P_1 - P_2\right) = \frac{V_{02}-A_2 x}{\beta_e}\dot{P}_2 + Q_2 \end{cases} \tag{2}$$

where C_{t} is the coefficient of the internal leakage of the cylinder, β_e is the effective bulk modulus of the hydraulic fluid in the container, V_{01}, V_{02} are the original total control volumes of the two cylinder chambers, respectively (including the volume of the servo valve, pipelines, and cylinder chambers), and Q_1 represents the supply flow rate to the forward chamber (or cylinder-end), Q_2 represents the return flow rate of the return chamber (or rod-end). Q_1 and Q_2 are given as [24]

$$\begin{cases} Q_1 = k_{q1}x_v\left[s(x_v)\sqrt{P_s - P_1} + s(-x_v)\sqrt{P_1 - P_r}\right] \\ Q_2 = k_{q2}x_v\left[s(x_v)\sqrt{P_2 - P_r} + s(-x_v)\sqrt{P_s - P_2}\right]. \end{cases} \tag{3}$$

Define function

$$s(*) = \begin{cases} 1, & \text{if } * \geqslant 0 \\ 0, & \text{if } * < 0 \end{cases}$$

$$k_{q1} = C_d w_1 \sqrt{\frac{2}{\rho}}$$

$$k_{q2} = C_d w_2 \sqrt{\frac{2}{\rho}} \tag{4}$$

where P_{s} is the supply pressure, P_r is the tank pressure, x_v is the spool displacement of the servo valve, k_{q1}, k_{q2} are the flow gain coefficients of the servo valve, C_d is the discharge coefficient, w_1 and w_2 are the spool valve area gradients, and ρ is the fluid density.

The effects of servo-valve dynamics have been included by some researchers [4], but this requires an additional sensor to obtain the spool position. Since only minimal performance improvement is achieved for position tracking, other researchers neglect servo-valve dynamics according to practice status, as in [12] and [15]. Since a high-response servo valve is used here, we assume that the control applied to the spool valve is directly proportional to the spool position, thus the following equation is given by $x_v = \psi u$, where ψ is a positive constant and u is input voltage. Hence, from (4), $s(x_v) = s(u)$ is obtained.

Therefore, (3) can be transformed to

$$\begin{cases} Q_1 = g_1 u[s(u)\sqrt{P_s - P_1} + s(-u)\sqrt{P_1 - P_r}] \\ Q_2 = g_2 u[s(u)\sqrt{P_2 - P_r} + s(-u)\sqrt{P_s - P_2}] \end{cases} \tag{5}$$

where $g_1 = k_{q1}\psi$ and $g_2 = k_{q2}\psi$.

Define the state variables as $x = [x_1, x_2 x_3 x_4]^T = [x \dot{x} P_1 P_2]^T$. The entire system, including (1), (2), and (5) can be expressed in a state- space form as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{m}[A_1 x_3 - A_2 x_4 - K x_1 - F_l(t)] \\ \dot{x}_3 &= \frac{\beta_e}{V_{01} + A_1 x_1}[-A_1 x_2 - C_t(x_3 - x_4) + g_1 u R_1(x)] \\ \dot{x}_4 &= \frac{\beta_e}{V_{02} - A_2 x_1}[A_2 x_2 + C_t(x_3 - x_4) - g_2 u R_2(x)] \end{aligned} \quad (6)$$

here

$$\begin{aligned} R_1(x) &= s(u)\sqrt{P_s - x_3} + s(-u)\sqrt{x_3 - P_r} \\ R_2(x) &= s(u)\sqrt{x_4 - P_r} + s(-u)\sqrt{P_s - x_4}. \end{aligned}$$

Given the desired motion trajectory $x_d(t)$, the objective of this paper is to design a bounded control input u so that the output x_1 tracks as closely as possible to the desired motion trajectory $x_d(t)$ in spite of various model uncertainties, including unknown nonlinear parameters. Based on the realistic hydraulic system, the following practical assumption is given as follows.

Assumption 1: The desired position $x_d(t)$, its velocity $\dot{x}_d(t)$, and acceleration $\ddot{x}_d(t)$ are all bounded.

III. Nonlinear Adaptive Robust Control With Unknown Nonlinear Parameters

A. Design Model and Issues to be Addressed

In order to simplify the state-space equation, firstly, define parameters set $\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^T$ as $\theta_1 = 1/m, \theta_2 = A_1\beta_e, \theta_3 = \beta_e C_t, \theta_4 = \beta_e g_1, \theta_5 = \beta_e g_2, \theta_6 = A_2\beta_e$; parameters set $\beta = [\beta_1, \beta_2]^T$ as $\beta_1 = V_{01}/A_1, \beta_2 = V_{02}/A_2$, and $d(t) = F_l(t)/m$.

In order to make system (6) fall into the classic class of systems known as strict feedback form to use the backstepping method appropriately, define a new state variable as $\bar{x}_3 = A_1 x_3 - A_2 x_4$. Then, the state-space equation (6) is transformed as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \theta_1(\bar{x}_3 - K x_1) - d(t) \\ \dot{\bar{x}}_3 &= \frac{1}{\beta_1 + x_1}[-\theta_2 x_2 - \theta_3(x_3 - x_4) + \theta_4 u R_1(x)] \\ &\quad - \frac{1}{\beta_2 - x_1}[\theta_6 x_2 + \theta_3(x_3 - x_4) - \theta_5 u R_2(x)]. \end{aligned} \quad (7)$$

From (7), it is clear that the nonlinear system state-space equation includes nonlinear parameters β_1, β_2. In most previous papers, researchers always treated β_1 and β_2 as known certain parameters. However, in fact, since the value of the original control volumes V_{01} and V_{02}, including the volume of pipelines, servo valve, and cylinder chambers, are very difficult to obtain precisely and may change as indicated in the discussion in the Introduction, and $\beta_1 = V_{01}/A_1, \beta_2 = V_{02}/A_2$, so parameters β_1 and β_2 are really uncertain.

Moreover, C_t, β_e, and C_d are variable in the whole work process due to different temperature and environments, etc., so these parameters are uncertain. Accurate spool valve area gradient w is difficult to be obtained and ρ may be variable under different conditions, so g_1 and g_2 are also uncertain. Sometimes the load mass m is variational, so m is also uncertain. Thus, linear parameters $\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^T$ are all uncertain. In addition, $d(t)$ is clearly an uncertain nonlinearity. However, the areas of two chambers and the effective bulk modulus of spring K can be obtained accurately, so they are treated as certain and known parameters in this paper.

Therefore, the uncertainties of nonlinear system (7) derive from uncertain nonlinear parameters set β, uncertain linear parameters set θ, and uncertain nonlinearities $d(t)$.

Since the existing nonlinear adaptive control methods based on conventional Lyapunov stability are invalid to the uncertain nonlinear parameters β_1 and β_2, most previous research assumed that V_{01} and V_{02}, that is, β_1 and β_2, are certain and known and only took into account uncertain linear parameters θ, such as [15] and [16], or both uncertain linear parameters θ and uncertain nonlinearities $d(t)$, such as [17] and [18]. Therefore, it is a challenging project to propose a nonlinear adaptive control method that can be used in an electro-hydraulic system with unknown nonlinear parameters β_1 and β_2.

In the following, by defining a special Lyapunov function, a nonlinear adaptive control and parameters adaptation scheme will be proposed to overcome the difficult problem from the unknown nonlinear parameters β_1 and β_2. Moreover, the adaptive robust control and adaptation laws for the whole system (7) compensating for all uncertain parameters and uncertain nonlinearities are presented by combining backstepping techniques and a simple robust method.

Though β_1 and β_2 are uncertain, in practical hydraulic systems, they are both bounded and positive. In fact, $\beta_1 > L_1$ and $\beta_2 > L_2$; in addition, $\theta, d(t)$ are also bounded. Thus, the following practical assumption is made.

Assumption 2:

1) $\beta_1 \leqslant B_1, \beta_2 \leqslant B_2$.

2) $\theta_i \in \Omega_{\theta_i} \triangleq [\theta_{i\min} \theta_{i\max}] i = 1, 2, \ldots, 6$, and $|d(t)| \leqslant D$.

L_1 is the possible maximal value of the absolute value of load negative displacement, L_2 is the possible maximal positive displacement of load, and B_1, B_2, and D are positive constants. Thus, we have

$$\begin{aligned} L_1 &< \beta_1 \leqslant B_1 \\ L_2 &< \beta_2 \leqslant B_2. \end{aligned} \quad (8)$$

B. Adaptive Control of a Class of Nonlinear Systems With Nonlinear Unknown Parameters

In order to solve the problem derived from unknown nonlinear parameters, first we will discuss the tracking problem of the following classical first-order nonlinear system with unknown nonlinear parameters like the third equation of system (7). This system is defined as

$$\dot{x} = \frac{\sigma_1^T f_{l1}(x) + \sigma_2^T f_{l2}(x) u}{\sigma_3^T f_{n0}(x)} \quad (9)$$

where $x \in R$ is the state, $u \in R$ is the control input, $f_{l1}(x) \in R^{n_1}, f_{l2}(x) \in R^{n_2}$., and $f_{n0}(x) \in R^{n_3}$ are all continuous functions and differentiable, and $\sigma_1 \in R^{n_1}, \sigma_2 \in R^{n_2}$, and $\sigma_3 \in R^{n_3}$ are all uncertain constant parameters sets in which σ_3 are clearly uncertain nonlinear parameters like β_1 and β_2 of system (7)

The control objective is to force x to track a desired trajectory x_d asymptotically.

Assumption 3: $\dot{x}_d$ is existent and bounded.

Assumption 4: $\sigma_i \in \Omega_{\sigma i} \triangleq [\sigma_{i\min} \sigma_{i\max}], i = 2, 3$ in which the sign of σ_3 is known, and $\sigma_2^T f_{l2}(x) \neq 0, \forall \sigma_2 \in \Omega_{\sigma 2}$; $\sigma_3^T f_{n0}(x) \neq 0, \forall \sigma_3 \in \Omega_{\sigma 3}$

Define $\hat{\sigma}_i$, denote the estimate of σ_i and $\tilde{\sigma}_i$, and denote the estimation errors, $\tilde{\sigma}_i = \sigma_i - \hat{\sigma}_i, i = 1, 2, 3$.

Let tracking error $e = x - x_d$. In order to construct the controller and adaptation law for system (9) with nonlinear parameterization, we develop a different special-type Lyapunov function, instead of the most widely used Lyapunov function, which is in conventional quadratic form. This Lyapunov function is designed as follows:

$$V_0 = \frac{1}{2} f(x) e^2. \tag{10}$$

The function $f(x)$ is defined as

$$f(x) = \sigma_3^T f_{n0}(x) \lambda(x) \tag{11}$$

where the function $\lambda(x)$ is satisfied as follows:

$$\begin{cases} 1) & \lambda(x) \text{is differentiable} \\ 2) & \text{Make sure that } f(x) > 0. \end{cases} \tag{12}$$

Thus, V_0 is a positive definite function with respect to tracking error e. In the following, based on the Lyapunov function V_0, we will derive a controller such that $\dot{V}_0 \leqslant 0$ to achieve asymptotic tracking control.

The time derivative of tracking error e along the system (9) is obtained as

$$\dot{e} = \dot{x} - \dot{x}_d = \frac{\sigma_1^T f_{l1}(x) + \sigma_2^T f_{l2}(x) u}{\sigma_3^T f_{n0}(x)} - \dot{x}_d. \tag{13}$$

Taking into account (10), (11), and (13), the time derivative of V_0 is obtained as

$$\begin{aligned} \dot{V}_0 &= \frac{1}{2} \sigma_3^T e^2 \frac{d\left[f_{n0}(x)\lambda(x)\right]}{dt} + \sigma_3^T f_{n0}(x) \lambda(x) e \dot{e} \\ &= \frac{1}{2} \sigma_3^T e^2 \left[\frac{d[f_{n0}(x)]}{dt} \lambda(x) + f_{n0}(x) \frac{d[\lambda(x)]}{dt} \right] \\ &\quad + \lambda(x) e \left[\sigma_1^T f_{l1}(x) + \sigma_2^T f_{l2}(x) u - \sigma_3^T f_{n0}(x) \dot{x}_d \right]. \end{aligned} \tag{14}$$

It is clear that the above equation does not include uncertain nonlinear parameters, that is, all of its uncertain parameters are linear. Thus, through defining the novel-type Lyapunov function as (10), the problem of solving unknown nonlinear parameters is converted to solve the problem of unknown linear parameters. Subsequently, we can use the conventional adaptive method to design a controller and adaptation law. Therefore, we can obtain the following results.

The controller is designed as

$$\begin{aligned} u = \frac{1}{\hat{\sigma}_2^T f_{l2}(x)} \Bigg\{ &-\hat{\sigma}_1^T f_{l1}(x) + \hat{\sigma}_3^T f_{n0}(x) \dot{x}_d - \frac{1}{\lambda(x)} k e \\ &- \frac{1}{2\lambda(x)} \hat{\sigma}_3^T e \overline{f(x)} \Bigg\} \end{aligned} \tag{15}$$

where $\overline{f(x)} = (d[f_{n0}(x)])/(dt)\lambda(x) + f_{n0}(x)(d[\lambda(x)])/(dt)$, k is a positive constant.

Substituting controller (15) into (14), we have

$$\begin{aligned} \dot{V}_0 = &-ke^2 + \frac{1}{2} \tilde{\sigma}_3^T e^2 \overline{f(x)} \\ &+ \lambda(x) e \left[\tilde{\sigma}_1^T f_{l1}(x) + \tilde{\sigma}_2^T f_{l2}(x) u - \tilde{\sigma}_3^T f_{n0}(x) \dot{x}_d \right]. \end{aligned} \tag{16}$$

In order to get the update law of parameters σ_1, σ_2, and σ_3, define the following Lyapunov function:

$$V_d = V_0 + \frac{1}{2} \sum_{i=1}^{3} \tilde{\sigma}_i^T T_i^{-1} \tilde{\sigma}_i$$

where $T_i, i = 1, 2, 3$ are positive-definite constant diagonal matrixes.

Taking into account (15), the time derivative of V_d is

$$\begin{aligned} \dot{V}_d &= \dot{V}_0 - \tilde{\sigma}_1^T \mathrm{T}_1^{-1} \dot{\hat{\sigma}}_1 - \tilde{\sigma}_2^T \mathrm{T}_2^{-1} \dot{\hat{\sigma}}_2 - \tilde{\sigma}_3^T \mathrm{T}_3^{-1} \dot{\hat{\sigma}}_3 \\ &= -ke^2 + \tilde{\sigma}_1^T \left[\lambda(x) e f_{l1}(x) - \mathrm{T}_1^{-1} \dot{\hat{\sigma}}_1 \right] \\ &\quad + \tilde{\sigma}_2^T \left[\lambda(x) e f_{l2}(x) u - \mathrm{T}_2^{-1} \dot{\hat{\sigma}}_2 \right] \\ &\quad + \frac{1}{2} \tilde{\sigma}_3^T e \left[e \overline{f(x)} - 2\lambda(x) f_{n0}(x) \dot{x}_d \right] - \tilde{\sigma}_3^T \mathrm{T}_3^{-1} \dot{\hat{\sigma}}_3. \end{aligned} \tag{17}$$

To make sure that $\dot{V}_d \leqslant 0$ and guarantee that $\hat{\sigma}_2^T f_{l2}(x) \neq 0$, the adaptation law is chosen as

$$\begin{aligned} \dot{\hat{\sigma}}_1 &= \mathrm{T}_1 \lambda(x) e f_{l1}(x) \\ \dot{\hat{\sigma}}_2 &= \mathrm{Pr}\, oj_{\sigma_2} \{ \mathrm{T}_2 \lambda(x) e f_{l2}(x) u \} \\ \dot{\hat{\sigma}}_3 &= \frac{1}{2} \mathrm{T}_3 e [\overline{f(x)} e - 2\lambda(x) f_{n0}(x) \dot{x}_d]. \end{aligned} \tag{18}$$

The function $\mathrm{Pr}\, oj_{\sigma_2}\{*\}$ is chosen to make sure that

$$\begin{aligned} &1) \hat{\sigma}_2^T f_{l2}(x) \neq 0 \\ &2) \tilde{\sigma}_2^T \left[\lambda(x) e f_{l2}(x) u - \mathrm{T}_2^{-1} \dot{\hat{\sigma}}_2 \right] \leqslant 0. \end{aligned} \tag{19}$$

Here, we simply chose function $\mathrm{Pr}\, oj_{\sigma_2}\{*\}$ as

$$\mathrm{proj}_{\sigma_2}\{*\} = \begin{cases} 0, & \text{if } \hat{\sigma}_2 = \sigma_{2\max} \text{ and } * > 0 \\ 0, & \text{if } \hat{\sigma}_2 = \sigma_{2\min} \text{ and } * < 0 \\ *, & \text{otherwise.} \end{cases} \tag{20}$$

In the following, we will verify that, if the function $\mathrm{Pr}\, oj_{\sigma_2}\{*\}$ is chosen as (20), the conditions (19) will be satisfied.

Proof: From (20), it is clear that $\sigma_2 \in \Omega_{\sigma_2} \triangleq [\sigma_{2\min} \sigma_{2\max}]$; thus from assumption 4, it is known that $\hat{\sigma}_2^T f_{l2}(x) \neq 0$ is guaranteed.

From (20) and adaptation law (18), it is also shown that, if $\hat{\sigma}_2 = \sigma_{2\max}$ and $\mathrm{T}_2 \lambda(x) e f_{l2}(x) u > 0$, we can get $\dot{\hat{\sigma}}_2 = 0$ and $\tilde{\sigma}_2 = \sigma_2 - \hat{\sigma}_2 = \sigma_2 - \sigma_{2\max} \leqslant 0$, thus we can obtain that $\tilde{\sigma}_2^T [\lambda(x) e f_{l2}(x) u - \mathrm{T}_2^{-1} \dot{\hat{\sigma}}_2] \leqslant 0$. If $\hat{\sigma}_2 = \sigma_{2\min}$ and $\mathrm{T}_2 \lambda(x) e f_{l2}(x) u < 0$, then we can get

$\dot{\hat{\sigma}}_2 = 0$ and $\tilde{\sigma}_2 = \sigma_2 - \hat{\sigma}_2 = \sigma_2 - \sigma_{2\min} \geqslant 0$, so we can obtain that $\tilde{\sigma}_2^T[\lambda(x)ef_{l2}(x)u - \mathrm{T}_2^{-1}\dot{\hat{\sigma}}_2] \leqslant 0$. In another case, it is known that $\dot{\hat{\sigma}}_2 = \mathrm{T}_2\lambda(x)ef_{l2}(x)u$, so $\tilde{\sigma}_2^T[\lambda(x)ef_{l2}(x)u - \mathrm{T}_2^{-1}\dot{\hat{\sigma}}_2] = 0$, therefore, we always can obtain $\tilde{\sigma}_2^T[\lambda(x)ef_{l2}(x)u - \mathrm{T}_2^{-1}\dot{\hat{\sigma}}_2] \leqslant 0$. Hence, the conditions (20) are satisfied.

Therefore, substituting the above adaptation law (18) into (17), we can have $\dot{V}_d \leqslant -ke^2 \leqslant 0$. Thus, it is easy to verify that the closed-loop system is globally stable and, when $t \to \infty$, $e \to 0$.

We summarize the above results into the following theorem.

Theorem 1: For the nonlinear system (9) consisting of uncertain nonlinear parameters and satisfying assumptions 3 and 4, under the controller (15) and adaptation laws (18), the closed-loop system is globally stable and asymptotic tracking is achieved, i.e., $\lim_{t\to\infty} x(t) = x_d(t)$.

Remark 1: It has been shown that the choice of the novel-type Lyapunov function (10) plays an important role in the controller design, with which the unknown nonlinear parameters can be converted into unknown linear parameters. Moreover, it is worth noting that, for a given system, a different function $\lambda\,(x)$ can be found to construct a different Lyapunov function V_0. Hence, the resulting controller is not unique, and the control performance is also affected by different choice of function $\lambda\,(x)$. In general, to make the controller much simpler, according to the feature of the practical system, the function $\lambda(x)$ should be as simple as possible as long as the conditions (12) are satisfied.

C. Controller Design of Electro-Hydraulic System

In the following, we will use a backstepping-like method to design the controller of the whole system, in which the above method will be used to solve the problem derived from the nonlinear uncertain parameters β_1 and β_2.

Step 1) Let $\hat{\theta}$ denote the estimate of θ and $\tilde{\theta}$ denote estimation errors, $\tilde{\theta} = \theta - \hat{\theta}$. Define the system output tracking error as $z_1 = x_1 - x_d(t)$. Then, we can have

$$\dot{z}_1 = x_2 - \dot{x}_d(t). \tag{21}$$

Here, treat x_2 as virtual control input, which is chosen as

$$x_{2d} = \dot{x}_d(t) - k_1 z_1 \tag{22}$$

where k_1 is a positive constant.

Define the error between the real value of x_2 and the virtual control $x_{2\mathrm{d}}$ as $z_2 = x_2 - x_{2d}$ and, combining (21) and (22), we can have

$$\dot{z}_1 = -k_1 z_1 + z_2. \tag{23}$$

It is clear that, if z_2 is small or converges to zero, the tracking error z_1 will be smaller or converge to zero, thus the next step is to make z_2 as small as possible.

Step 2) In this step, $\bar{x}_3$ is treated as virtual control, which is synthesized such that z_2 is as small as possible. As follows, the time derivatives of z_2 along the system (7) is obtained as

$$\begin{aligned}\dot{z}_2 &= \dot{x}_2 - \dot{x}_{2d} = \theta_1(\bar{x}_3 - Kx_1) - d(t) - \dot{x}_{2d}\\ &= \hat{\theta}_1(\bar{x}_3 - Kx_1) + \tilde{\theta}_1(\bar{x}_3 - Kx_1) - d(t) - \dot{x}_{2d}\end{aligned} \tag{24}$$

where, according to (22) and (23), we have

$$\dot{x}_{2d} = \ddot{x}_d(t) - k_1[x_2 - \dot{x}_d(t)].$$

A controller consisting of adaptive control and robust control is designed as

$$\begin{aligned}&\alpha = \alpha_a + \alpha_{\mathrm{ar}}\\ &\alpha_a = Kx_1 + \frac{1}{\hat{\theta}_1}\dot{x}_{2d}, \alpha_{\mathrm{ar}} = \frac{1}{\hat{\theta}_1}a_r\end{aligned} \tag{25}$$

where α_a is an adaptive control law and α_{ar} is a hybrid control law including adaptation parameter and robust controller α_r, in which α_r is used to compensate for the unmodeled uncertainties coming from d (t). Usually, a variable structure control (VSC) law is adopted to solve the kind of problem from $d\ (t)$, but the controller obtained by VSC is not differentiable, thus it cannot be used in the next step design, since it is necessary that any virtual controller in each step must be differentiable in the backstepping method. Therefore, VSC cannot be adopted here, so we present a simple method to overcome the problem. Robust controller α_r is designed simply as

$$\alpha_r = -k_2 z_2 - z_1, \quad k_2 = D\delta^{-1} + \eta \tag{26}$$

where δ and η are both positive constants, and δ is the tracking error bound of z_2, that is, $|z_2|$ converges to δ in finite time, which will be expatiated later.

Let $z_3 = \bar{x}_3 - \alpha$, taking into account (24), (25), and (26). Then, we can get

$$\dot{z}_2 = \tilde{\theta}_1(\alpha - Kx_1) - \left[\frac{D}{\delta}z_2 + d(t)\right] - \eta z_2 - z_1 + \theta_1 z_3. \tag{27}$$

Define the Lyapunov function as

$$V_2 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}\tilde{\theta}^T\Gamma_1^{-1}\tilde{\theta}$$

where Γ_1 is a positive-definite constant adaptation rate diagonal matrix.

Thus, the time derivative of V_2 is

$$\dot{V}_2 = z_1\dot{z}_1 + z_2\dot{z}_2 - \tilde{\theta}^T\Gamma_1^{-1}\dot{\hat{\theta}}. \tag{28}$$

Substituting (23) and (27) into (28), we can have

$$\begin{aligned}\dot{V}_2 = &- k_1 z_1^2 - \eta z_2^2 + z_2\tilde{\theta}_1(\alpha - Kx_1)\\ &- \left[\frac{D}{\delta}z_2^2 + d(t)z_2\right] + \theta_1 z_2 z_3 - \tilde{\theta}^T\Gamma_1^{-1}\dot{\hat{\theta}}.\end{aligned} \tag{29}$$

The virtual adaptation algorithm of this step is chosen as

$$\dot{\hat{\theta}} = \gamma = \Gamma_1 z_2[\alpha - Kx_1, 0, 0, 0, 0, 0]^T. \tag{30}$$

Substituting the virtual adaptation algorithm (30) into (29), we can get

$$\dot{V}_2 = -k_1 z_1^2 - \eta z_2^2 - \left[\frac{D}{\delta}z_2^2 + d(t)z_2\right] + \theta_1 z_2 z_3. \tag{31}$$

Thus, it can be deduced that, if $|z_2| \geqslant \delta$, then $(1)/(\delta)z_2^2 \geqslant |z_2|$, and, from assumption 2, we know $|d(t)| \leqslant D$, therefore, it is always true that $(D)/(\delta)z_2^2 + d(t)z_2 \geqslant 0$. Thus, in this case, it is shown that, if $z_3 = 0$, then we will always obtain that $\dot{V}_2 = -k_1 z_1^2 - \eta z_2^2 - [D\delta^{-1}z_2^2 + d(t)z_2] \leqslant 0$. Therefore, it is easy to verify that $|z_2|$ converges to within bound δ in finite time.

Thus, the following step is just to make z_3 converge to zero or as small as possible.

Step 3) Rewrite the last differential equation of system (7) as

$$\begin{aligned}\dot{x}_3 &= \frac{1}{\beta_1 + x_1}[-\theta_2 x_2 - \theta_3(x_3 - x_4) + \theta_4 u R_1] \\ &\quad - \frac{1}{\beta_2 - x_1}[\theta_6 x_2 + \theta_3(x_3 - x_4) - \theta_5 u R_2] \\ &= \frac{f_l(x)}{f_n(x)} \end{aligned} \tag{32}$$

where

$$\begin{aligned} f_n(x) &= (\beta_1 + x_1)(\beta_2 - x_1) \\ &= -x_1^2 + (\beta_2 - \beta_1)x_1 + \beta_1\beta_2 \\ f_l &= -(\beta_2\theta_2 + \beta_1\theta_6)x_2 \\ &\quad - \theta_3(\beta_1 + \beta_2)(x_3 - x_4) + (\theta_2 - \theta_6)x_1 x_2 \\ &\quad + [\theta_5\beta_1 R_2 + \theta_4\beta_2 R_1 + (\theta_5 R_2 - \theta_4 R_1)x_1]u. \end{aligned}$$

The above differential equation consists of uncertain nonlinear parameters β_1 and β_2, and the adaptive method of step 2) cannot be used, so most research treats β_1 and β_2 as certain known parameters. However, in practice, β_1 and β_2 are both unknown, as mentioned above,

In the following, for subsystem (32) with uncertain nonlinear parameters, we will use the method developed in Section III-B to derive the controller and update laws of uncertain parameters (including uncertain nonlinear and linear parameters) to make sure that the tracking error z3 of this step converges to zero or is as small as possible.

From Section II, we know x_1 is the displacement of load, and from assumption 2 and (8), it is clear that $\beta_1 + x_1 > 0$ and $\beta_2 - x_1 > 0$, so we can get

$$(\beta_1 + x_1)(\beta_2 - x_1) > 0.$$

Thus, according to Section III-B, $\lambda(x) = 1$ is chosen simply, and then we have

$$\begin{aligned} f(x) &= (\beta_1 + x_1)(\beta_2 - x_1) \\ &= \beta_1\beta_2 + (\beta_2 - \beta_1)\, x_1 - x_1^2. \end{aligned}$$

So, we define the following Lyapunov function as

$$V_3 = \frac{1}{2}\left[\beta_1\beta_2 + (\beta_2 - \beta_1)x_1 - x_1^2\right] z_3^2.$$

The time derivative of V_3 along system (7) is given by

$$\begin{aligned} \dot{V}_3 &= \frac{1}{2}(\beta_2 - \beta_1)x_2 z_3^2 - x_1 x_2 z_3^2 \\ &\quad + z_3\left[\beta_1\beta_2 + (\beta_2 - \beta_1)x_1 - x_1^2\right]\dot{z}_3 \\ &= \frac{1}{2}(\beta_2 - \beta_1)x_2 z_3^2 - x_1 x_2 z_3^2 \\ &\quad + z_3[-(\beta_2\theta_2 + \beta_1\theta_6)x_2 \\ &\qquad - (\beta_1 + \beta_2)\theta_3(x_3 - x_4) + (\theta_2 - \theta_6)x_1 x_2] \\ &\quad + z_3[\theta_5\beta_1 R_2 + \theta_4\beta_2 R_1 + (\theta_5 R_2 - \theta_4 R_1)x_1]u \\ &\quad - z_3\left[\beta_1\beta_2 + (\beta_2 - \beta_1)x_1 - x_1^2\right]\dot{\alpha} \end{aligned} \tag{33}$$

where $\dot{\alpha}$ can be obtained from (25), (26) and system (7), giving

$$\dot{\alpha} = \frac{\partial\alpha}{\partial x_1}x_2 + \frac{\partial\alpha}{\partial x_2}\dot{x}_2 + \frac{\partial\alpha}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 + \sum_{i-1}^{3}\frac{\partial\alpha}{\partial x_d^{(i-1)}}x_d^{(i)}. \tag{34}$$

Let

$$\dot{\alpha}_1 = \frac{\partial\alpha}{\partial x_1}x_2 + \frac{\partial\alpha}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 + \sum_{i=1}^{3}\frac{\partial\alpha}{\partial x_d^{(i-1)}}x_d^{(i)} \tag{35}$$

$$\dot{\alpha}_2 = \frac{\partial\alpha}{\partial x_2}\dot{x}_2 = \frac{\partial\alpha}{\partial x_2}[\theta_1(\bar{x}_3 - Kx_1) - d(t)]. \tag{36}$$

Then, $\dot{\alpha} = \dot{\alpha}_1 + \dot{\alpha}_2$, in which $\dot{\alpha}_1$ is certain, but, since θ_1 and $d(t)$ are uncertain, $\dot{\alpha}_2$ is also uncertain.

Define unknown parameters set $\varepsilon = [\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6, \varepsilon_7]^T$ as $\varepsilon_1 = \theta_5\beta_1, \varepsilon_2 = \theta_4\beta_2, \varepsilon_3 = \beta_2\theta_2 + \beta_1\theta_6, \varepsilon_4 = (\beta_1 + \beta_2)\theta_3, \varepsilon_5 = \theta_2 - \theta_6, \varepsilon_6 = \beta_1\beta_2\theta_1, \varepsilon_7 = (\beta_2 - \beta_1)\theta_1$ and $\varphi = [\varphi_1\varphi_2]^T$ as $\varphi_1 = \beta_1\beta_2, \varphi_2 = \beta_2 - \beta_1$, Taking into account the realistic hydraulic system, we give the following practical assumption as follows.

Assumption 5:

1) $\varepsilon_j \in \Omega_{\varepsilon_j} \triangleq [\varepsilon_{j\,\min}\varepsilon_{j\,\max}] j = 1, 2.$
2) $\varepsilon_1 R_2 + \varepsilon_2 R_1 + (\theta_5 R_2 - \theta_4 R_1)x_1 \neq 0, \forall\theta_4 \in \Omega_{\theta_4}, \theta_5 \in \Omega_{\theta_5}$ and $\varepsilon_1 \in \Omega_{\varepsilon_1}\varepsilon_2 \in \Omega_{\varepsilon_2}$.

Also, from Assumption 2, we can get

$$|\varphi_1| \leqslant B_1 B_2, \quad |\varphi_2| < \max(B_1, B_2). \tag{37}$$

Substituting (34)–(34)–(36) and the parameters ε, φ into (33), the following equation can be obtained:

$$\begin{aligned} \dot{V}_3 &= \frac{1}{2}\varphi_2 x_2 z_3^2 - x_1 x_2 z_3^2 - z_3\left[\varphi_1 + \varphi_2 x_1 - x_1^2\right]\dot{\alpha}_1 \\ &\quad + z_3[\varepsilon_1 R_2 + \varepsilon_2 R_1 + (\theta_5 R_2 - \theta_4 R_1)x_1]u \\ &\quad + z_3[-\varepsilon_3 x_2 - \varepsilon_4(x_3 - x_4) + \varepsilon_5 x_1 x_2] \\ &\quad - z_3(\bar{x}_3 - Kx_1)\frac{\partial\alpha}{\partial x_2}(\varepsilon_6 + \varepsilon_7 x_1) \\ &\quad + z_3\frac{\partial\alpha}{\partial x_2}x_1^2[\theta_1(\bar{x}_3 - Kx_1)] \\ &\quad + z_3\frac{\partial\alpha}{\partial x_2}[\varphi_1 + \varphi_2 x_1 - x_1^2]d(t). \end{aligned} \tag{38}$$

$\hat{\varepsilon}_i$ denotes the estimate of $\varepsilon_i, i = 1, 2, \ldots, 7$ and $\hat{\varphi}_j$ denotes the estimate of $\varphi_j, j = 1, 2$. Thus, the controller is designed as

$$\begin{aligned} u &= \frac{u_1 + u_2 + u_3}{\hat{\varepsilon}_1 R_2 + \hat{\varepsilon}_2 R_1 + (\hat{\theta}_5 R_2 - \hat{\theta}_4 R_1) x_1} \\ u_1 &= x_1(x_2 z_3 - x_1 \dot{\alpha}_1) \\ u_2 &= \hat{\varepsilon}_3 x_2 + \hat{\varepsilon}_4 (x_3 - x_4) - \hat{\varepsilon}_5 x_1 x_2 \\ &\quad + (\hat{\varphi}_1 + \hat{\varphi}_2 x_1)\dot{\alpha}_1 - \frac{1}{2}\hat{\varphi}_2 x_2 z_3 \\ &\quad + \frac{\partial \alpha}{\partial x_2}(\bar{x}_3 - K x_1)(\hat{\varepsilon}_6 + \hat{\varepsilon}_7 x_1) \\ &\quad - \frac{\partial \alpha}{\partial x_2} x_1^2 [\hat{\theta}_1 (\bar{x}_3 - K x_1)] - \hat{\theta}_1 z_2. \end{aligned} \tag{39}$$

Taking into account (37), we can design the robust controller u_3 as

$$\begin{aligned} u_3 = &- k_3 z_3 - \mathrm{Dsgn}(z_3) \\ &\times [B_1 B_2 + \max(B_1, B_2)\sqrt{x_1^2 + \varsigma_2}] \sqrt{\left(\frac{\partial \alpha}{\partial x_2}\right)^2 + \varsigma_1}. \end{aligned}$$

where u_1 is used to compensate for the certain known items, u_2 is an adaptive controller, which is used to overcome the problem from uncertain parameters, u_3 is a robust controller which is used to compensate the unknown item related to $d\,(t)$, and $k_3, \varsigma_1, \varsigma_2$ are positive constants.

Remark 2: In practice, R_1 and R_2 are both seldom zero when the system is operating smoothly, since P_1 and P_2 are seldom close toP_s and P_r. In the rare case that R_1 and R_2 equal zero (e.g., due to the noise in P_1 and P_2), it is set to a small positive number to avoid the problem of dividing by zero.

Thus, according to adaptation (18), and to make sure that the virtual control α of step 2 is well defined, combining with (30), the adaptation law is given by

$$\dot{\hat{\theta}} = \mathrm{Pr}\,oj_{\theta_i}\left\{\gamma + \Gamma_1 z_3 \begin{bmatrix} z_2 + \frac{\partial \alpha}{\partial x_2} x_1^2 (\bar{x}_3 - K x_1) \\ 0 \\ 0 \\ -x_1 R_1 u \\ x_1 R_2 u \\ 0 \end{bmatrix}\right\} \quad i = 1, 2, \ldots, 6 \tag{40}$$

$$\dot{\hat{\varepsilon}}_a = \mathrm{Pr}\,oj_{\varepsilon_j}\{\Gamma_2 z_3 u [R_2, R_1]^T\} \quad j = 1, 2 \tag{41}$$

$$\begin{aligned} \dot{\hat{\varepsilon}}_b = \Gamma_3 z_3 \Big[&-x_2, (x_4 - x_3), x_1 x_2, \\ &- \frac{\partial \alpha}{\partial x_2}(\bar{x}_3 - K x_1), \frac{\partial \alpha}{\partial x_2}(\bar{x}_3 - K x_1) x_1\Big]^T \end{aligned} \tag{42}$$

$$\dot{\hat{\varphi}} = z_3 \Gamma_4 \left[z_3 \dot{\alpha}_1, \frac{1}{2} x_2 z_3 - x_1 \dot{\alpha}_1\right] \tag{43}$$

where $\varepsilon_a = [\varepsilon_1 \varepsilon_2]^T, \varepsilon_b = [\varepsilon_3 \varepsilon_4 \varepsilon_5 \varepsilon_6 \varepsilon_7]^T$, and $\hat{\varepsilon}_a, \hat{\varepsilon}_b$ denote the estimate of $\varepsilon_a, \varepsilon_b$, respectively. Γ_2, Γ_3, and Γ_4 are positive-definite constant adaptation rate diagonal matrixes.

Theorem 2: For the nonlinear electro-hydraulic system (7) with unknown nonlinear parameters β_1 and β_2 and Assumptions 1, 2, and 5 being satisfied, if adopt the controller (39), and use the adaptation law shown by (40)–(43), the following will be given.

1) In general, the tracking error z_1 and the transformed states $z = [z_1, z_2, z_3]^T$ and estimated parameters $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5, \hat{\theta}_6]^T, \hat{\varepsilon} = [\hat{\varepsilon}_1, \hat{\varepsilon}_2, \hat{\varepsilon}_3, \hat{\varepsilon}_4, \hat{\varepsilon}_5, \hat{\varepsilon}_6, \hat{\varepsilon}_7]^T \hat{\varphi} = [\hat{\varphi}_1, \hat{\varphi}_2]^T$, are all bound.
2) The maximal absolute value of the tracking error is smaller than δ/k_1, at $t \to \infty$, that is, $\lim_{t\to\infty}|z_1| \leqslant \delta/k_1$

Proof: Define the Lyapunov function as

$$V = V_2 + V_3 + \frac{1}{2}\tilde{\varepsilon}_a^T \Gamma_2^{-1} \tilde{\varepsilon}_a + \frac{1}{2}\tilde{\varepsilon}_b^T \Gamma_3^{-1} \tilde{\varepsilon}_b + \frac{1}{2}\tilde{\varphi}^T \Gamma_4^{-1} \tilde{\varphi}$$

where $\tilde{\varepsilon}_a = \varepsilon_a - \hat{\varepsilon}_a, \tilde{\varepsilon}_b = \varepsilon_b - \hat{\varepsilon}_b, \tilde{\varphi} = \varphi - \hat{\varphi}$ Thus, the time derivative of V is

$$\dot{V} = \dot{V}_2 + \dot{V}_3 - \tilde{\varepsilon}_a^T \Gamma_2^{-1} \dot{\hat{\varepsilon}}_a - \tilde{\varepsilon}_b^T \Gamma_2^{-1} \dot{\hat{\varepsilon}}_b - \tilde{\varphi}^T \Gamma_3^{-1} \dot{\hat{\varphi}}.$$

Substitute (29), (38), the controller (39), and adaptation law (40)–(43) into the above equation, and from Theorem 1, we can get

$$\begin{aligned} \dot{V} \leq &- k_1 z_1^2 - \eta z_2^2 - k_3 z_3^2 \\ &- \left[\frac{D}{\delta} z_2^2 + d(t) z_2\right] \\ &- \Big\{ D|z_3| \left[B_1 B_2 + \max(B_1, B_2)\sqrt{x_1^2 + \varsigma_2}\right] \\ &\quad \times \sqrt{(\partial \alpha / \partial x_2)^2 + \varsigma_1} \\ &\quad - \frac{\partial \alpha}{\partial x_2}\left[\beta_1 \beta_2 + (\beta_2 - \beta_1) x_1 - x_1^2\right] d(t) z_3 \Big\}. \end{aligned} \tag{44}$$

From Assumption 2 and (37), we have

$$\begin{aligned} D|z_3| &\left[B_1 B_2 + \max(B_1, B_2)\sqrt{x_1^2 + \varsigma_2}\right] \sqrt{\left(\frac{\partial \alpha}{\partial x_2}\right)^2 + \varsigma_1} \\ &- \frac{\partial \alpha}{\partial x_2}\left[\beta_1 \beta_2 + (\beta_2 - \beta_1) x_1 - x_1^2\right] d(t) z_3 \geqslant 0 \end{aligned} \tag{45}$$

$$-\frac{D}{\delta} z_2^2 - d(t) z_2 \leqslant \frac{\delta d^2(t)}{4D} \leqslant \frac{\delta D}{4}. \tag{46}$$

Combining (44)–(46) leads to

$$\begin{aligned} \dot{V} &\leqslant -k_1 z_1^2 - \eta z_2^2 - k_3 z_3^2 - \frac{D}{\delta} z_2^2 - d(t) z \\ &\leqslant -k_1 z_1^2 - \eta z_2^2 - k_3 z_3^2 + \frac{\delta D}{4} \end{aligned}$$

where $\delta D/4$ is a known positive constant, so V is bounded together with the states $z = [z_1, z_2, z_3]^T$, and $\tilde{\theta}, \tilde{\varepsilon}_a, \tilde{\varepsilon}_b \tilde{\varphi}$ are all bounded, and since $\theta, \varepsilon_a, \varepsilon_b \varphi$ are all bounded too, so $\hat{\theta} = [\hat{\theta}_1 \hat{\theta}_2 \, \hat{\theta}_3 \hat{\theta}_4 \hat{\theta}_5 \hat{\theta}_6]^T, \hat{\varepsilon} = [\hat{\varepsilon}_a \hat{\varepsilon}_b]^T = [\hat{\varepsilon}_1 \hat{\varepsilon}_2 \hat{\varepsilon}_3 \hat{\varepsilon}_4 \hat{\varepsilon}_5 \hat{\varepsilon}_6 \hat{\varepsilon}_7]^T, \hat{\varphi} = [\hat{\varphi}_1 \hat{\varphi}_2]^T$ are all bounded. Thus, 1) of Theorem 2 is proved.

Moreover, since $D \geqslant |d(t)|$, it is obtained that if $|z_2| \geqslant \delta$, then $(D)/(\delta) z_2^2 + d(t) z_2 \geqslant 0$, so $\dot{V} \leqslant 0$, then leads to $|z_2|$ converges to within bounded δ, that is, $|z_2| \leqslant \delta$, in finite time. From (23), $\dot{z}_1 + k_1 z_1 = z_2$ is obtained.

As [25], define

$$y = z_1 e^{k_1 t}. \tag{47}$$

So, $\dot{y} = (\dot{z}_1 + k_1 z_1) e^{k_1 t} = z_2 e^{k_1 t}$ and we have $-\delta e^{k_1 t} \leqslant \dot{y} \leqslant \delta e^{k_1 t}$

Integrating the above equation, we can get

$$y(t) \leqslant \frac{\delta}{k_1} e^{k_1 t} - \frac{\delta}{k_1} + y(0).$$

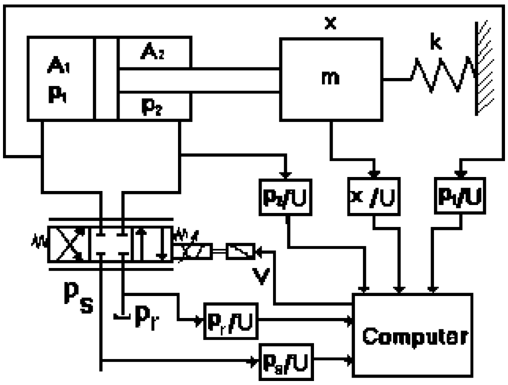

Fig. 2. Experimental setup.

From (47), we can get

$$z_1 = y(t)e^{-k_1 t} \leqslant \frac{\delta}{k_1} + \left[y(0) - \frac{\delta}{k_1}\right] e^{-k_1 t}.$$

Thus

$$\lim_{t\to\infty} z_1 \leqslant \lim_{t\to\infty} \left\{\frac{\delta}{k_1} + \left[y(0) - \frac{\delta}{k_1}\right] e^{-k_1 t}\right\} = \frac{\delta}{k_1}.$$

Similarly, we can obtain $\lim_{t\to\infty} z_1 \geqslant -\delta/k_1$, hence, $\lim_{t\to\infty}|z_1| \leqslant \delta/k_1$ is obtained. Thus, 2) of Theorem 2 is proved.

Remark 3: It is seen that the value of the system tracking error is related to controller parameter δ and k_1, which can be chosen arbitrarily. In particular, when a certain k_1 is chosen, the smaller δ is, the smaller tracking error will be. However, on the other hand, when δ is smaller, the performance of the control system will be affected; for example, the chatting phenomena will be made with δ being smaller. Therefore, it is important to select an appropriate δ.

Remark 4: Although we obtained that z_1, z_2, z_3 are all bounded, from which we can easily prove that the position x_1, the velocity, x_2, and $\bar{x}_3 = A_1 x_3 - A_2 x_4$ are bounded, it cannot guarantee that x_3 and x_4 are bounded. However, in a realistic hydraulic system, when the system is operating smoothly, x_3, x_4, which imply the pressure in the two cylinder chambers P_1 and P_2, respectively, are always smaller than the system pressure P_s and bigger than the tank pressure P_r, that is, $P_r < x_3 < P_s, P_r < x_4 < P_s$, where$P_s$ andP_r are both positive constants. So, x_3 and x_4 are both bounded, and therefore all of the state variables of system (6) are bounded, thus the closed-loop system is globally stable.

IV. Experiments

A. Experiment Setup

To test the proposed nonlinear adaptive robust control strategy and study fundamental problems associated with the control of electro-hydraulic systems, in Fig. 2, the experimental system installation of the model is presented. In this installation, the servo valve used is HRV-Servo solenoid valve made by Bosch Company, of which the control input is $[-10\ 10]$ V. Dimensions of the cylinder are 40 mm/22 mm/300 mm. The pump used is a gear pump also made by Bosch Company, of which displacement is 25 cm^3/rev. The supply pressure P s is set at 60 bar by a relief valve.

The system states used in the controller, including load position x and the pressures in two cylinder chambers P_1 and P_2, are directly measured by position and pressure transducers, respectively, and the load velocity x_2 is differential from position x with a PC, which is also implementation of the digital control of system in real time. In addition, in order to improve the precision of control system, the supply pressure P_s and the tank pressureP_r are also directly measured by pressure transducers, respectively. Also, all of the analog measurement signals (the cylinder position x_1, pressure P_1, P_2, P_s, and P_r) are fed back to the PC through a plugged-in 16-bit A/D and D/A board. To attenuate the influence of noise, all measured signals are processed through a low-pass filter.

The estimated system parameters that are used as the original values of the proposed nonlinear adaptive robust controller are set as follows: the effective bulk modulus of spring $K = 7000$ N/m, which is treated as a constant parameter during the control process; load mass $m = 100$ kg; the effective bulk modulus of the hydraulic fluid $\beta e = 2000$ Mpa; the coefficient of internal leakage of the cylinder $C_t = 2 \times 10^{-15}$(m^{3}s^{-1}/Pa); $g_1 = g_2 = 3.5 \times 10^{-8}\text{m}^3 s^{-1}/(\text{V}\sqrt{P_a})$. Set the original position of piston at the end of cylinder, at which the original total control volume of the two cylinder chambers is $V_1 = 1.2 \times 10^{-4}\text{m}^3$ and $V_2 = 2.5 \times 10^{-4}\text{m}^3$.

The controller parameters are designed as

$$\begin{aligned}
D =& 2.5\\
\delta =& 0.1, B_1 = B_2 = 0.7\\
k_1 =& k_3 = \eta = 20\\
\varsigma_1 =& \varsigma_2 = 0.01\\
\Gamma_1 =& \text{diag}\{4 \times 10^{-8}, 0, 0, 10^{-2}, 10^{-2}, 0\}\\
\Gamma_2 =& \text{diag}\{10^3, 10^3, 10^5, 10^{-7}, 10^{-2}, 10^{-8}, 10^{-7}\}\\
\Gamma_3 =& \text{diag}\{10^{-5}, 10^{-4}\}.
\end{aligned}$$

Here, in order to show the influence of the uncertain parameters, specify uncertain nonlinear parameters, and, to test the performance of the proposed control scheme, three controllers are used and compared, among which the first is just the proposed nonlinear adaptive robust control (ARC-NP), and the second is the same control law as the proposed control but without using parameter adaptation, that is, let $\Gamma_1 = \text{diag}\{0,0,0,0,0,0\}, \Gamma_2 = \text{diag}\{0,0,0,0,0,0,0\}$, and $\Gamma_3 = \text{diag}\{0,0\}$.

The third controller uses the adaptive method but without considering the uncertain control volume (ARC-LP), that is, the nonlinear parameters β_1 and β_2 are tread as known constants, of which steps 1) and 2) are the same, and step) 3 is different. The controller u' is given by

$$\begin{aligned}
u' &= \frac{u_1' + u_2'}{\hat{\theta}_4 R_1 h_1(x) + \hat{\theta}_5 R_2 h_2(x)}\\
u_1' &= \hat{\theta}_2 x_2 h_1(x) + \hat{\theta}_3 (x_3 - x_4)[h_1(x) + h_2(x)]\\
&\quad + \hat{\theta}_6 x_2 h_2(x) + \hat{\theta}_1 \left[\frac{\partial \alpha}{\partial x_2}(\bar{x}_3 - K x_1) - z_2\right]\\
u_2' &= -k_3 z_3 + \dot{\alpha}_1 - \text{Dsgn}(z_3)\sqrt{\left(\frac{\partial \alpha}{\partial x_2}\right)^2 + \varsigma_1}.
\end{aligned}$$

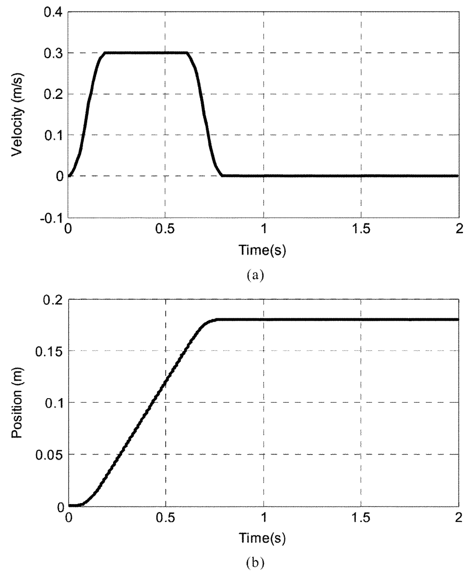

Fig. 3. First desired trajectories. (a) Desired velocity. (b) Desired position.

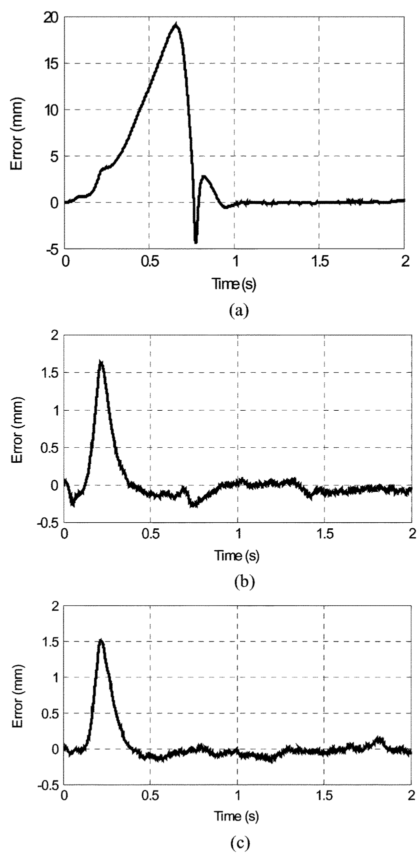

Fig. 4. Tracking errors with no change. (a) Without parameter adaptations. (b) ARC-LP. (c) Proposed control (ARC-NP).

The adaptation law is chosen as

$$\dot{\hat{\theta}} = \mathrm{Pr}\,oj_{\theta_i}\left\{\gamma + \mathrm{N}z_3 \begin{bmatrix} z_2 - \frac{\partial\alpha}{\partial x_2}(\bar{x}_3 - Kx_1) \\ -x_2 h_1(x) \\ -(x_3 - x_4)[h_1(x) + h_2(x)] \\ h_1(x)R_1 u \\ h_2(x)R_2 u \\ -x_2 h_2(x) \end{bmatrix}\right\}, \quad i = 1, 2, \ldots, 6$$

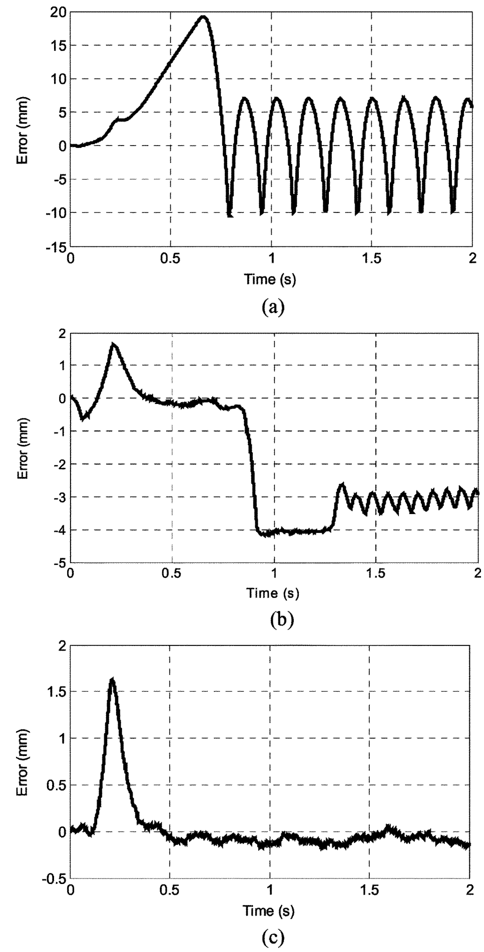

Fig. 5. Tracking errors with V_{01}, V_{02} changes. (a) Without parameter adaptations. (b) ARC-LP. (c) Proposed control (ARC-NP).

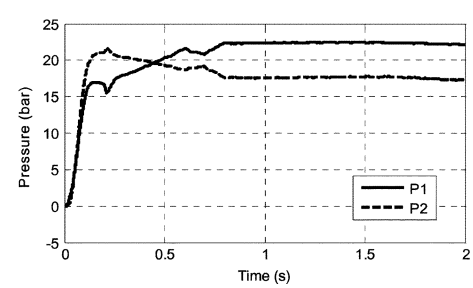

Fig. 6. Pressures in two cylinder chambers with V_{01}, V_{02} changes.

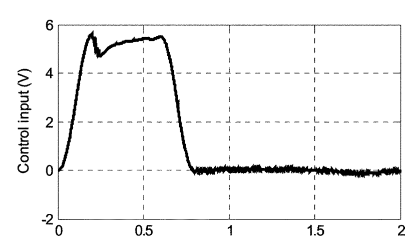

Fig. 7. Control input with V_{01}, V_{02} changes.

where $h_1(x) = (1)/(\beta_1 + x_1), h_2(x) = (1)/(\beta_2 - x_1)$, N is the positive constant adaptation-rate diagonal matrix, the value of N is selected as $\mathrm{diag}\{4 \times 10^{-8}, 10^4, 10^{-5}, 10^5, 10^5, 10^3\}$, and the remaining parameters are the same as given above.

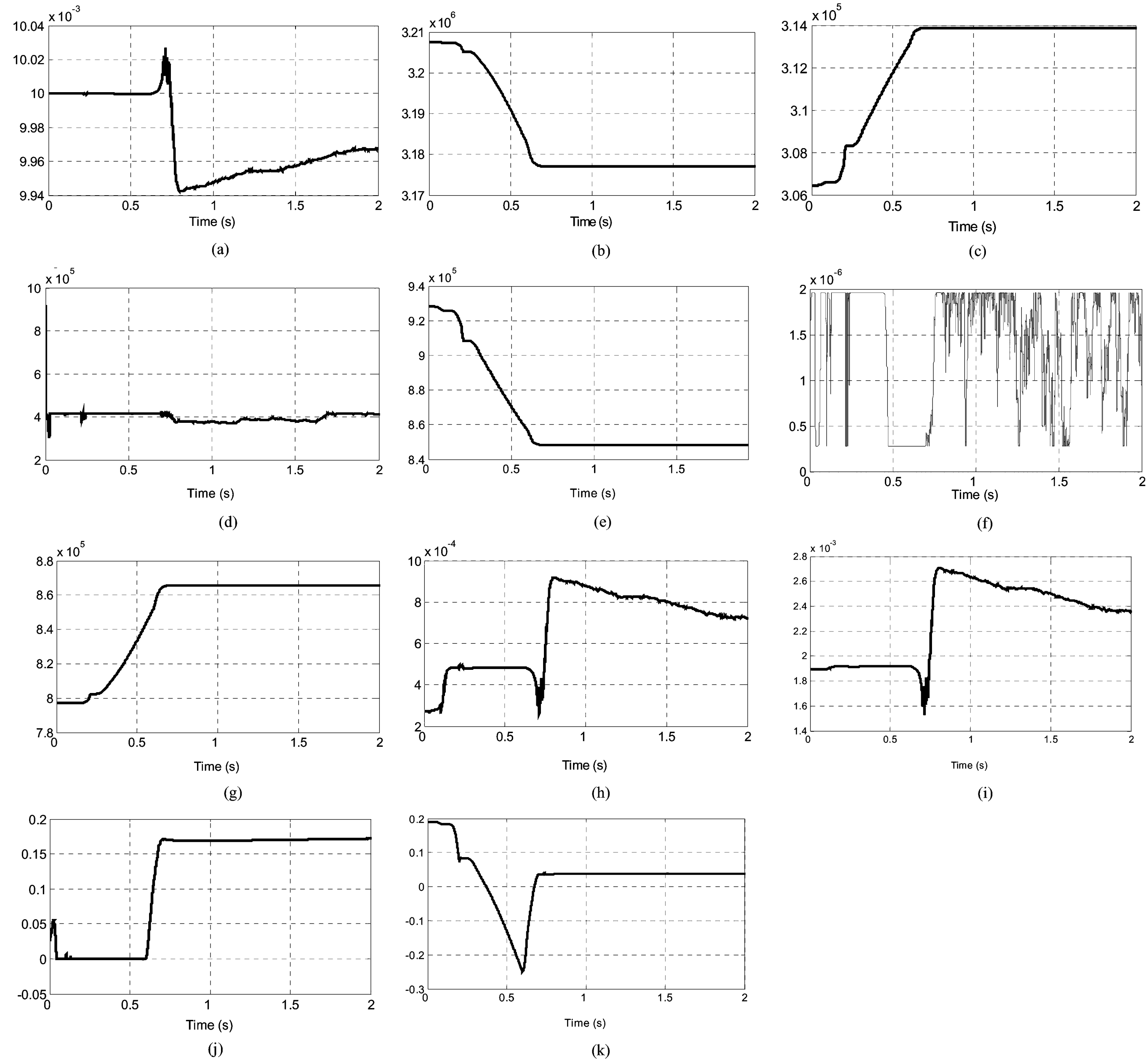

Fig. 8. Parameters estimations. (a) $\hat{\theta}_1$. (b) $\hat{\theta}_4$ and $\hat{\theta}_5$. (c) $\hat{\varepsilon}_1$. (d) $\hat{\varepsilon}_2$. (e) $\hat{\varepsilon}_3$. (f) $\hat{\varepsilon}_4$. (g) $\hat{\varepsilon}_5$. (h) $\hat{\varepsilon}_6$. (i) $\hat{\varepsilon}_7$. (j) $\hat{\varphi}_1$. (k) $\hat{\varphi}_2$.

B. Experimental Results

The three controllers first track a desired motion trajectory shown in Fig. 3 as in [18], which has a maximum velocity of 0.3 m/s and an acceleration of 3 m/s^2. Fig. 4 shows the tracking errors when the system parameters are not changed. As seen, ADC-NP control and ADC-LP control can both obtain better control performance, but the controller without adaptation laws cannot obtain good performance, of which the error is too large to be accepted, since the selected original values of parameters are different from the real values of the system. This illustrates the effectiveness of using parameter adaptation.

To test the influence of variations of nonlinear parameters β_1 and β_2 on the performance of the control system, we increase the values of the original total control volumes V_{01} and V_{02} by increasing the length of pipelines between the servo valve and cylinder approximately one time to change the values of β_1 and β_2. The tracking errors are shown in Fig. 5, from which it is clear that the tracking error obtained by the proposed controller has almost no changes, but, when the adaptation laws are not used and the adaptation laws do not consider the nonlinear parameters β_1 and β_2, their tracking errors increase significantly during the last periods, and the oscillatory phenomena are made. It is illuminated that the differential values of the original total control volumes can significantly affect the performance of the hydraulic control system, and the proposed adaptive robust control strategy can eliminate the effects. Thus, we have shown that the objective of this paper is achieved.

The transient pressures P_1 and P_2 are shown in Fig. 6, and the control input is shown in Fig. 7. It can be seen that they

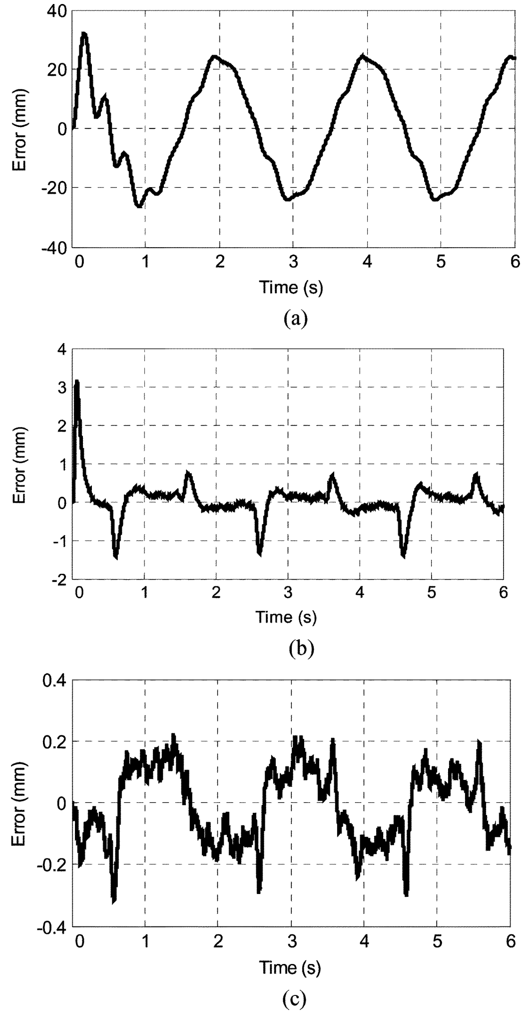

Fig. 9. Tracking errors for sine-wave motion with V_{01}, V_{02} changes. (a) Without parameter adaptations. (b) ARC-LP. (c) Proposed control (ARC-NP).

are all bounded and in the permissible capacity scope. All of the estimations of parameters are shown in Fig. 8; it can be seen that they are all bounded, and it is their update during the control process that improves the performance of the hydraulic system.

Second, the reference signal is a 0.5-Hz sinusoidal motion $x_d = 0.1\sin(\pi t)$ m. In order to show the influence of the variational value of the original total control volumes, we put the original piston position at the middle of cylinder and reset design parameter $\delta = 1$. The tracking errors are shown in Fig. 9. It is seen that, when the adaptation laws are not used, the track error is the largest, and the error of ADC-LP is much bigger than that of ADC-NP, specifically at the direction of position changes, which again illustrates the effectiveness of using nonlinear parameter adaptation. The pressures P_1, P_2 are shown in Fig. 10, and it is clear that they are bounded. The control input is shown in Fig. 11, which is also bounded.

V. Conclusion

A nonlinear adaptive robust controller has been presented for an electro-hydraulic system driven by a single-rod actuator with uncertain nonlinear parameters. Both the control structure and

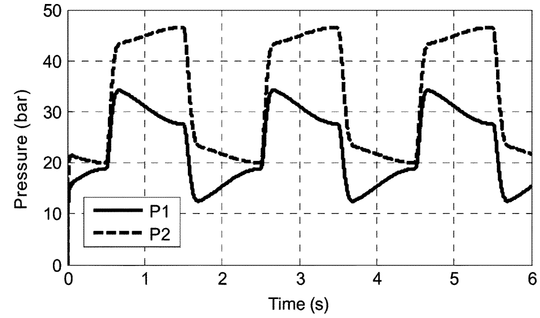

Fig. 10. Pressures in two cylinder chambers for sinusoidal motion with V_{01}, V_{02} changes.

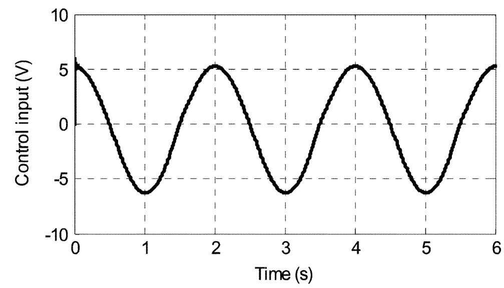

Fig. 11. Control input for sine-wave motion with V_{01}, V_{02} changes.

unknown nonlinear parameters tuning algorithms are developed through the newly chosen Lyapunov function. The whole system controller and adaptation schemes are given by combining backstepping techniques and a simple robust control method, which can compensate for all system uncertainties from the unknown nonlinear parameters, the unknown linear parameters, and uncertain nonlinearities. The experimental results show that the proposed control method and adaptation schemes can obtain a good performance when the position signal trajectories are tracked, even if the uncertain nonlinear parameters due to the varieties of the original control volumes exist. The main contribution of this paper is that a nonlinear adaptive control method has been presented to compensate for a class of uncertain nonlinear parameters of electro-hydraulic systems.

References

[1] J. W. Raade and H. Kazerooni, “Analysis and design of a novel hydraulic power source for mobile robots,” *IEEE Trans. Autom. Sci. Eng.*, vol. 2, no. 3, pp. 226–232, Jul. 2005.

[2] M. C. Wu and M. C. Shih, “Simulated and experimental study of hydraulic anti-lock braking system using sliding-mode PWM control,” *Mechatron.*, vol. 13, pp. 331–351, 2003.

[3] S. Daohang, B. B. Vladimir, and Y. Huayong, “New model and sliding mode control of hydraulic elevator velocity tracking system,” *Simul. Practice Theory*, vol. 9, no. 6, pp. 365–385, 2002.

[4] A. Alleyne and J. K. Hedrick, “Nonlinear adaptive control of active suspensions,” *IEEE Trans. Control Syst. Technol.*, vol. 3, no. 1, pp. 94–101, Mar. 1995.

[5] J. E. Bobrow and K. Lum, “Adaptive, high bandwidth control of a hydraulic actuator,” *ASME J. Dynam. Syst., Meas., Contr.*, vol. 118, no. 4, pp. 714–720, 1996.

[6] A. R. Plummer and N. D. Vaughan, “Robust adaptive control for hydraulic servosystems,” *ASME J. Dynam. Syst., Meas., Contr.*, vol. 118, no. 2, pp. 237–244, 1996.

[7] D. Li and S. E. Salcuden, “Modeling, simulation, and control of a hydraulic stewart platform,” in *Proc. IEEE Int. Conf. Robot. Autom.*, Apr. 1997, pp. 3360–3366.

[8] I. S. Yun and H. S. Cho, "Adaptive model following control of electrohydraulic velocity control system," *Proc. Inst. Elect. Eng.*, vol. 135, no. 2, pp. 149–156, Mar. 1988.
[9] G. A. Shol and J. E. Bobrow, "Experimental and simulations on the nonlinear control of a hydraulic servo system," *IEEE Trans. Control Syst. Technol.*, vol. 7, no. 1, pp. 238–247, Mar. 1999.
[10] R. Vossoughi and M. Donath, "Dynamic feedback linearization forelectro-hydraulically actuated control systems," *ASME J. Dynam. Syst. Meas., Contr.*, vol. 117, no. 4, pp. 468–477, 1995.
[11] L. D. Re and A. Isidori, "Performance enhancement of nonlinear drives by feedback linearization of linear-bilinear cascade models," *IEEE Trans. Control Syst. Technol.*, vol. 3, no. 2, pp. 299–308, May 1995.
[12] Y. Liu and H. Handroos, "Technical note sliding mode control for a class of hydraulic position servo," *Mechatron.*, vol. 9, no. 1, pp. 111–123, 1999.
[13] L. Hong, T. C. Kil, S. N. Tae, and Y. S. Yeong, "Vehicle longitudinal brake control using variable parameter sliding control," *Control Eng. Practice*, vol. 11, no. 4, pp. 403–411, 2003.
[14] D. Hisseine, "Robust tracking control for a hydraulic actuation system," in *Proc. IEEE Conf. Control Appl.*, Aug. 28–31, 2005, pp. 422–427.
[15] D. Garagic and K. Srinivasan, "Application of nonlinear adaptive control techniques to an electrohydraulic velocity servomechanism," *IEEE Trans. Control Syst. Technol.*, vol. 12, no. 2, pp. 303–314, Mar. 2004.
[16] A. Alleyne and R. Liu, "A simplified approach to force control for electro-hydraulic systems," *Control Eng. Practice*, vol. 8, pp. 1347–1356, 2000.
[17] R. Liu and A. Alleyne, "Nonlinear force/pressure tracking of an electro-hydraulic actuator," *ASME J. Dynam. Syst., Meas., Contr.*, vol. 122, pp. 232–237, 2000.
[18] B. Yao, F. Bu, and G. T. C. Chiu, "Nonlinear adaptive robust control of electro-hydraulic servo systems with discontinuous projections," in *Proc. IEEE Conf. Decision Control*, 1998, pp. 2265–2270.
[19] B. Yao, F. Bu, J. Reedy, and G. T. C. Chiu, "Adaptive robust motion control of single-rod hydraulic actuators: Theory and experiments," *IEEE/ASME Trans. Mechatron.*, vol. 5, no. 1, pp. 79–91, Mar. 2000.
[20] F. Bu and B. Yao, "Desired compensation adaptive robust control of single-rod electro-hydraulic actuator," in *Proc. Amer. Control Conf.*, Arlington, VA, 2001, pp. 3927–3931.
[21] S. Liu and B. Yao, "Indirect adaptive robust control of electro-hydraulic systems driven by single-rod hydraulic actuator," in *Proc. IEEE/ASME Int. Conf. Adv. Intell. MeChalroniCS*, 2003, pp. 296–301.
[22] S. Duraiswamy and G. T.-C. Chiu, "Nonlinear adaptive nonsmooth dynamic surface control of electro-hydraulic systems," in *Proc. Amer. Control Conf.*, Denver, CO, Jun. 2003, pp. 3287–3292.
[23] W. H. Zhu and J. C. Piedboeuf, "Adaptive output force tracking control of hydraulic cylinders with applications to robot manipulators," *ASME J. Dynam. Syst. Meas., Contr.*, vol. 127, pp. 206–217, 2005.
[24] H. E. Merritt, *Hydraulic Control Systems*. New York: Wiley, 1967.
[25] M. Bouri and D. Thomasset, "Sliding control of an electropneumatic actuator using an integral switching surface," *IEEE Trans. Control Syst. Technol.*, vol. 9, no. 2, pp. 368–375, Mar. 2001.

Cheng Guan received the B.Eng. degree, the M.Eng. degree in mechanical engineering, and the Ph.D. degree in electrical engineering all from Zhejiang University, Hangzhou, China, in 1991, 2002, and 2005, respectively.

He is currently a Postdoctoral Researcher with the Mechanical Design Institute, Zhejiang University. His research interests include design and control of electromechanical/electrohydraulic systems, adaptive and robust control, and nonlinear control.

Shuangxia Pan received the B.Eng. degree in mechanical engineering from Zhejiang University of Technology, Hangzhou, China, in 1984, and the M.Eng. degree and the Ph.D. degree in mechanical engineering from Zhejiang University, Hangzhou, China, in 1989 and 1992, respectively.

He is currently a Professor with the Mechanical Design Institute, Zhejiang University. His research interests are design and control of electromechanical systems modeling and fault detection and diagnostics.

Prof. Pan is currently Vice Secretary General of China Excavator Association and Vice Secretary General of the Mechanical Engineering Association of Zhejiang Province, China.