Brief Papers

Adaptive Control for Nonlinearly Parameterized Uncertainties in Robot Manipulators

N. V. Q. Hung, *Member, IEEE*, H. D. Tuan, *Member, IEEE*, T. Narikiyo, and P. Apkarian

***Abstract*—In this brief, a new adaptive control framework to compensate for uncertain nonlinear parameters in robot manipulators is developed. The designed adaptive controllers possess a linear parameter structure, guarantee global boundedness of the closed-loop system as well as tracking of a given trajectory within any prescribed accuracy. Our design approach takes advantage of a Lipschitzian property with respect to the plant nonlinear parameters. The outcome is that a very broad class of nonlinearly parameterized adaptive control problems for robot manipulators can be solved using this technique. Another feature of the proposed method is the design of low-dimensional estimator, even 1-D if desired, independently of the unknown parameter vector dimension. Simulations and experiments in friction compensation task for low-velocity tracking of a 2 degree-of-freedom planar robot demonstrate the viability of the technique and emphasize its advantages relatively to more classical approaches.**

***Index Terms*—Adaptive control, friction compensation, motion control, nonlinearities, parameter estimation, robot control, uncertain systems.**

I. Introduction

THE ORIGINAL and popular adaptive control theory usually deals with linear parameterizations (LP) of uncertainties, that is, it is assumed that uncertain quantities in dynamic systems are expressed linearly with respect to unknown parameters. Actually, most developed approaches such as gradient-based ones or recursive least squares [2], [11] rely heavily on this assumption. In the literature of robot control, most adaptive control techniques exploit the linear structure of manipulator dynamics [3] and effective techniques have been proposed in this context [11].

However, nonlinear parameterizations (NP) are very common in practical robot manipulators. A typical example is the Stribeck effect of frictional forces at joints of the manipulators [1]. Such an effect results in undesired tracking errors, especially at the low-velocity tracking task [8]. Adaptive controls for robot manipulators (see [9] and [12] for a survey) cannot successfully compensate for the Stribeck effect since they are based on the LP structure of unknown parameters. Also, most of adaptive friction compensation schemes in the literature of motion control only deal with either frictions with LP structure [5] or linearized models at the nominal values of the Stribeck friction parameters [7]. Recently, a Lyapunov-based adaptive control has been designed to compensate for the Stribeck effect under set-point control [4]. There are very few results in the literature addressing the adaptive control problem for NP of robot manipulators in a general manner.

In this brief, we exploit the recent results of our work [13] to formulate a general framework of adaptive control to compensate for uncertain nonlinear parameters appearing in robot manipulators. The proposed approach is applicable to any NP under Lipschitzian conditions. These conditions are satisfied for a broad class of practical systems. It is worth noticing that Lipschitzian parameterizations include as special cases convex/concave and smooth parameterizations. From the viewpoint of adaptive control for NP, we also redesign the traditional adaptive control for LP. The resulting adaptive control incorporates estimators of minimum dimension (1-D) independently of the parameter dimension.

The organization of this brief is as follows. In Section II, we discuss dynamic model of robot manipulators in the presence of nonlinearly parameterized uncertainties. Some results of adaptive controls for NP in [13] will be recalled at this stage. As main contributions of this brief, adaptive controllers for robot manipulators are synthesized in Section III. A general framework of adaptive control for NP in the system is developed first. Then, adaptive control with 1-D estimators is derived. In Section IV, an application of our adaptive control for friction compensation in tracking problem of a 2 degree-of-freedom (DOF) planar robot is introduced together with comparative simulations and experiments. Some concluding remarks are given in Section V.

This brief is the journal version of the conference paper [6].

II. Problem Statement

The dynamic model of a robot manipulator can be described by the following equation:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{f}_N(\mathbf{x},\boldsymbol{\theta}) = \boldsymbol{\tau}(t) \qquad (1)$$

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N. V. Q. Hung is with the Technical Research Center, Toyota Motor Corporation, Shizuoka 410-1193, Japan (e-mail: nvq_hung@ieee.org).

H. D. Tuan is with the School of Electrical Engineering and Telecommunications, The University of New South Wales, Sydney, NSW 2052, Australia (e-mail: h.d.tuan@unsw.edu.au).

T. Narikiyo is with the Department of Electrical and Computer Engineering, Toyota Technological Institute, Nagoya 468-8511, Japan (e-mail: n-tatsuo@toyota-ti.ac.jp).

P. Apkarian is with ONERA-CERT, 31055 Toulouse, France (e-mail: apkarian@cert.fr).

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where $\mathbf{q}(t) \in R^n$ is the joint coordinates of the manipulator, $\boldsymbol{\tau} \in R^n$ is the torque applied to the joints, $\mathbf{H}(q) \in R^{n\times n}$ is the symmetric positive definite inertia matrix of the links, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) \in R^{n\times n}$ is a matrix representing Coriolis and centrifugal effects, $\mathbf{g}(\mathbf{q}) \in R^n$ is the gravitational torques, and $\mathbf{f}_N(\mathbf{x},\boldsymbol{\theta}) \in R^n$ represents dynamics whose constant or slowly-varying uncertain parameter $\boldsymbol{\theta}$ appears nonlinearly in the system. Note that $\mathbf{x}$ can be any component of the system state, for instance $\mathbf{x} = [\mathbf{q}^T, \dot{\mathbf{q}}^T]^T$.

We focus on the case where the uncertainties admit a general multiplicative form, i.e.,

$$\begin{aligned}\mathbf{f}_N(\mathbf{x},\boldsymbol{\theta}) &= [f_{N1}(\mathbf{x},\boldsymbol{\theta}_1),\dots,f_{Nn}(\mathbf{x},\boldsymbol{\theta}_n)]^T\\ f_{Ni}(\mathbf{x},\boldsymbol{\theta}_i) &= \mathbf{g}_i(\mathbf{x},\boldsymbol{\theta}_i)\mathbf{h}_i(\mathbf{x},\boldsymbol{\theta}_i), \quad i=1,\dots,n.\end{aligned} \tag{2}$$

Here i stands for the ith joint of the manipulator and functions $\mathbf{g}_i(\mathbf{x},\boldsymbol{\theta}_i)$, $\mathbf{h}_i(\mathbf{x},\boldsymbol{\theta}_i)$ are assumed nonlinear and Lipschitzian in $\boldsymbol{\theta}_i$, $\boldsymbol{\theta}_i = [\theta_{i1},\dots,\theta_{ip_i}]^T \in R^{p_i}$. As it will be discussed later, a typical example of uncertainty admitting this form is the Stribeck effect of frictional forces in joints of robot manipulators [1].

The Lipschitzian condition for functions is recalled first.

Definition 1: Functions $\mathbf{g}_i(\mathbf{x},\boldsymbol{\theta}_i) : R^r \times R^{p_i} \to (R^{m_i})^T$ and $\mathbf{h}_i(\mathbf{x},\boldsymbol{\theta}_i) : R^r \times R^{p_i} \to R^{m_i}$ are said to be Lipschitzian in $\boldsymbol{\theta}_i$ if there exist continuous functions $L_{ij}(\mathbf{x}) \ge 0, \ell_{ij}(\mathbf{x}) \ge 0$ such that the following inequalities:

$$\begin{aligned}\|\mathbf{g}_i(\mathbf{x},\boldsymbol{\theta}_i) - \mathbf{g}_i(\mathbf{x},\bar{\boldsymbol{\theta}}_i)\| &\le \sum_{j=1}^{p_i} L_{ij}(\mathbf{x})|\theta_{ij} - \bar{\theta}_{ij}| \ \forall(\mathbf{x},\boldsymbol{\theta}_i,\bar{\boldsymbol{\theta}}_i)\\ \|\mathbf{h}_i(\mathbf{x},\boldsymbol{\theta}_i) - \mathbf{h}_i(\mathbf{x},\bar{\boldsymbol{\theta}}_i)\| &\le \sum_{j=1}^{p_i} \ell_{ij}(\mathbf{x})|\theta_{ij} - \bar{\theta}_{ij}| \ \forall(\mathbf{x},\boldsymbol{\theta}_i,\bar{\boldsymbol{\theta}}_i)\end{aligned}$$

hold true.

Here and after, $\|\cdot\|$ denotes the standard Euclidean norm. Note that all smooth or convex/concave functions satisfy the previously mentioned Lipschitz condition.

Next, for system (1), the following properties are very important [3].

Property 2.1: The inertia matrix $\mathbf{H}(\mathbf{q})$ is positive definite and satisfies $\lambda_{\min}\mathbf{I} \le \mathbf{H}(\mathbf{q}) \le \lambda_{\max}\mathbf{I}$, with $0 < \lambda_{\min} < \lambda_{\max} < \infty$, where $\lambda_{\min}$, $\lambda_{\max}$ are minimal and maximal eigenvalues of $\mathbf{H}(\mathbf{q})$.

Property 2.2: The matrix $\dot{\mathbf{H}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is skew-symmetric.

Property 2.3: The sum of the first three terms in the LHS of (1) are expressed linearly with respect to a suitable set of constant dynamic parameters

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\mathbf{a} \tag{3}$$

where $\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}}) \in R^{n\times\omega}$ is a nonlinear matrix function and $\mathbf{a} \in R^\omega$ is a vector of unknown dynamic parameters.

The following result of [13] will be frequently used in subsequent developments.

Lemma 1: Given Lipschitzian functions $\mathbf{g}_i(\mathbf{x},\boldsymbol{\theta}_i)$, $\mathbf{h}_i(\mathbf{x},\boldsymbol{\theta}_i)$, let $L_i(\mathbf{x})$ and $\ell_i(\mathbf{x})$ be defined as

$$\begin{aligned}L_i(\mathbf{x}) &:= \max_{j=1,2,\dots,p} L_{ij}(\mathbf{x})\\ \ell_i(\mathbf{x}) &:= \max_{j=1,2,\dots,p} \ell_{ij}(\mathbf{x})\end{aligned} \tag{4}$$

then, for $\boldsymbol{\theta}_i \in R_+^{p_i}$, the following inequalities:

$$\begin{aligned}&e(t)\mathbf{g}_i(\mathbf{x},\boldsymbol{\theta}_i)\mathbf{h}_i(\mathbf{x},\boldsymbol{\theta}_i)\\ &\quad\le e(t)\mathbf{g}_i(\mathbf{x},0)\mathbf{h}_i(\mathbf{x},0) + |e(t)|\\ &\qquad\times\left\{L_i(\mathbf{x})\ell_i(\mathbf{x})\left(\sum_{j=1}^{p_i}\theta_{ij}\right)^2\right.\\ &\qquad\quad + [\|\mathbf{h}_i(\mathbf{x},0)\|\,L_i(\mathbf{x}) + \|\mathbf{g}_i(\mathbf{x},0)\|\,\ell_i(\mathbf{x})]\\ &\qquad\quad\left.\times\sum_{j=1}^{p_i}\theta_{ij}\right\}\end{aligned} \tag{5}$$

hold true for any $e(t) \in R$.

Proof: See Appendix A.

Our goal is to control the rigid manipulator to track a given trajectory $\mathbf{q}_d(t)$ by designing a nonlinear adaptive control to compensate for all uncertainties which are either LP uncertain dynamics according to Property 2.3 or NP as defined by (2), in system (1). For simplicity of the derivations throughout the brief, it is assumed that $\boldsymbol{\theta}_i \in R_+^{p_i}$, i.e., $\theta_{ij} \ge 0, j = 1,2,3,\dots,p_i$. At the end of Section III-B, we will see that the general case $\boldsymbol{\theta}_i \in R^{p_i}$ can be easily retrieved from our results. While traditional adaptive controls can be effectively applied only in the context of LP [11], Lemma 1 reveals an ability to approximate the NP by its certain part plus a part of LP. We will use the key property (5) to design a novel nonlinear adaptive control for the system. □

III. Main Results

Define vector $\mathbf{s}(t) \in R^n$ as a "velocity error" term

$$\mathbf{s}(t) = \dot{\tilde{\mathbf{q}}}(t) + \boldsymbol{\Lambda}\tilde{\mathbf{q}}(t) = \dot{\mathbf{q}}(t) - \dot{\mathbf{q}}_r(t) \tag{6}$$

where $\boldsymbol{\Lambda} = \mathrm{diag}[\lambda_1,\lambda_2,\dots,\lambda_n] \in R^{n\times n}$ is an arbitrary positive definite matrix, $\tilde{\mathbf{q}}(t) = \mathbf{q}(t) - \mathbf{q}_d(t)$ is the position tracking error, and $\dot{\mathbf{q}}_r(t) = \dot{\mathbf{q}}_d(t) - \boldsymbol{\Lambda}\tilde{\mathbf{q}}(t)$, is called the "reference velocity." According to Property 2.3, the dynamics of the system (1) can be rewritten in terms of the "velocity error" $s(t)$ as

$$\begin{aligned}&\mathbf{H}(\mathbf{q})\dot{\mathbf{s}}(t) + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\mathbf{s}(t)\\ &\qquad = \boldsymbol{\tau}(t) - \mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r)\mathbf{a} - \mathbf{f}_N(\mathbf{x},\boldsymbol{\theta})\end{aligned} \tag{7}$$

with the identity $\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r)\mathbf{a} = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}_r + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_r + \mathbf{g}(\mathbf{q})$ used.

By definition (6), the tracking error $\tilde{q}_i(t)$ obtained from $s_i(t)$ through the previous designed first-order low-pass filter is

$$\tilde{q}_i(t) = \tilde{q}_i(t_0)e^{-\lambda_i(t-t_0)} + \int_{t_0}^{t} s_i(\zeta)e^{\lambda_i(\zeta-t)}d\zeta$$

where $\tilde{q}_i(t_0)$ is the tracking error of joint ith of the robot manipulator at the time t_0. If $|s_i(t)| \le \rho, \forall t \ge t_0$, then

$$\begin{aligned}|\tilde{q}_i(t)| &\le |\tilde{q}_i(t_0)|e^{-\lambda_i(t-t_0)} + \int_{t_0}^{t} |s_i(\zeta)|e^{\lambda_i(\zeta - t)}d\zeta \\ &\le \left(|\tilde{q}_i(t_0)| - \frac{\rho}{\lambda_i}\right)e^{-\lambda_i(t-t_0)} + \frac{\rho}{\lambda_i}.\end{aligned} \tag{8}$$

The relation (8) means that $\lim_{t\to\infty}|\tilde{q}_i(t)| \le (\rho/\lambda_i)$ whenever $\lim_{t\to\infty}|s_i(t)| \le \rho$. Therefore, in the next development, the model (7) is used for designing a control input $\tau(t)$ which guarantees the velocity error $s(t) \to 0$ under LP uncertainty a and NP uncertainty θ. As shown before, such performance of $s(t)$ ensures the convergence to 0 of tracking error $\tilde{q}(t)$ when $t \to \infty$.

A. Discontinuous Adaptive Control Design

Consider a quadratic Lyapunov function candidate

$$V_1(t) := \frac{1}{2}\mathbf{s}^T(t)\mathbf{H}(\mathbf{q})\mathbf{s}(t).$$

By Property 2.2, its time derivative can be written as

$$\begin{aligned}\dot{V}_1(t) &= \mathbf{s}^T\left(\boldsymbol{\tau} - \mathbf{Ya} - \mathbf{f}_N(\mathbf{x}, \boldsymbol{\theta})\right) \\ &= \mathbf{s}^T(\boldsymbol{\tau} - \mathbf{Ya}) - \sum_{i=1}^{n} s_i f_{Ni}(\mathbf{x}, \boldsymbol{\theta}_i)\end{aligned}$$

where the notations on $t, \mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r$ are neglected for simplicity. In view of relation (5), it follows that

$$\begin{aligned}\dot{V}_1(t) \le\ & \mathbf{s}^T(\boldsymbol{\tau} - \mathbf{Ya}) + \left(\sum_{i=1}^{n} s_i \mathbf{g}_i(\mathbf{x},0)\mathbf{h}_i(\mathbf{x},0)\right) \\ &+ \sum_{i=1}^{n}|s_i|\left\{L_i(\mathbf{x})\ell_i(\mathbf{x})\left(\sum_{j=1}^{p_i}\theta_{ij}\right)^2 \right. \\ &\quad + \left[\|\mathbf{h}_i(\mathbf{x},0)\|L_i(\mathbf{x}) + \|\mathbf{g}_i(\mathbf{x},0)\|\ell_i(\mathbf{x})\right] \\ &\quad \left. \times \sum_{j=1}^{p_i}\theta_{ij}\right\}.\end{aligned} \tag{9}$$

With the definitions

$$\begin{aligned}\mathbf{W}(\mathbf{x}) &:= \mathrm{diag}[\mathbf{w}_1(\mathbf{x}), \mathbf{w}_2(\mathbf{x}), \ldots, \mathbf{w}_n(\mathbf{x})] \in R^{n\times 2n} \\ \boldsymbol{\Phi}(\mathbf{s}, \mathbf{x}) &:= \mathrm{diag}[\mathrm{sgn}(s_1)\mathbf{w}_1(\mathbf{x}), \ldots, \mathrm{sgn}(s_n)\mathbf{w}_n(\mathbf{x})] \in R^{n\times 2n} \\ \boldsymbol{\beta} &:= \begin{bmatrix}\boldsymbol{\beta}_1^T & \boldsymbol{\beta}_2^T & \cdots & \boldsymbol{\beta}_n^T\end{bmatrix}^T \in R^{2n} \\ \mathbf{w}_i(\mathbf{x}) &= [w_{i1} \quad w_{i2}] \\ &:= [L_i(\mathbf{x})\ell_i(\mathbf{x}) \quad \|\mathbf{h}_i(\mathbf{x},0)\|L_i(\mathbf{x}) + \|\mathbf{g}_i(\mathbf{x},0)\|\ell_i(\mathbf{x})] \\ \boldsymbol{\beta}_i &= [\beta_{i1} \quad \beta_{i2}]^T \\ &:= \left[\left(\sum_{j=1}^{p_i}\theta_{ij}\right)^2 \quad \sum_{j=1}^{p_i}\theta_{ij}\right]^T\end{aligned} \tag{10}$$

the inequality (9) can be rewritten as

$$\dot{V}_1(t) \le \mathbf{s}^T(\boldsymbol{\tau} - \mathbf{Ya}) + \mathbf{s}^T\mathbf{f}_N(\mathbf{x},0) + \mathbf{s}^T\boldsymbol{\Phi}(\mathbf{s},\mathbf{x})\boldsymbol{\beta}. \tag{11}$$

Therefore, the control input

$$\boldsymbol{\tau} = -\mathbf{K}_D\mathbf{s} + \mathbf{Y}\hat{\mathbf{a}} - \mathbf{f}_N(\mathbf{x},0) - \boldsymbol{\Phi}(\mathbf{s},\mathbf{x})\hat{\boldsymbol{\beta}} \tag{12}$$

results in

$$\dot{V}_1(t) \le -\mathbf{s}^T\mathbf{K}_D\mathbf{s} + \mathbf{s}^T[\mathbf{Y}\tilde{\mathbf{a}} - \boldsymbol{\Phi}(\mathbf{s},\mathbf{x})]\tilde{\boldsymbol{\beta}} \tag{13}$$

where $\tilde{\mathbf{a}} = \hat{\mathbf{a}} - \mathbf{a}$ and $\tilde{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}} - \boldsymbol{\beta}$ are parameter errors and $\mathbf{K}_D \in R^{n\times n}$ is an arbitrary positive definite matrix.

To derive update laws for the parameter estimates, we employ the following Lyapunov function:

$$V(t) = V_1(t) + \frac{1}{2}\left(\tilde{\mathbf{a}}^T\boldsymbol{\Gamma}_a^{-1}\tilde{\mathbf{a}} + \tilde{\boldsymbol{\beta}}^T\boldsymbol{\Gamma}_\beta^{-1}\tilde{\boldsymbol{\beta}}\right) \tag{14}$$

where $\boldsymbol{\Gamma}_a, \boldsymbol{\Gamma}_\beta$ are arbitrary positive definite matrices. It follows from (13) that

$$\dot{V}(t) \le -\mathbf{s}^T\mathbf{K}_D\mathbf{s} + \mathbf{s}^T[\mathbf{Y}\tilde{\mathbf{a}} - \boldsymbol{\Phi}(\mathbf{s},\mathbf{x})]\tilde{\boldsymbol{\beta}} + \dot{\hat{\mathbf{a}}}^T\boldsymbol{\Gamma}_a^{-1}\tilde{\mathbf{a}} + \dot{\hat{\boldsymbol{\beta}}}^T\boldsymbol{\Gamma}_\beta^{-1}\tilde{\boldsymbol{\beta}}. \tag{15}$$

Therefore, the following update laws:

$$\begin{aligned}\dot{\hat{\mathbf{a}}} &= -\boldsymbol{\Gamma}_a\mathbf{Y}^{\mathbf{T}}\mathbf{s} \\ \dot{\hat{\boldsymbol{\beta}}} &= \boldsymbol{\Gamma}_\beta\mathbf{W}^T(\mathbf{x})|\mathbf{s}| \\ |\mathbf{s}| &= [|s_1| \quad |s_2| \quad \cdots \quad |s_n|]^T\end{aligned} \tag{16}$$

yield

$$\dot{V}(t) \le -\mathbf{s}^T\mathbf{K}_D\mathbf{s}. \tag{17}$$

The last inequality implies that $V(t)$ is decreasing, and thus is bounded by $V(0)$. Consequently, $\mathbf{s}(t)$ and $\tilde{\mathbf{a}}(t), \tilde{\boldsymbol{\theta}}(t)$ must be bounded quantities by virtue of definition (14). Given the boundedness of the reference trajectory $\mathbf{q}_d, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d$, one has $\dot{\mathbf{s}}(t) \in L_\infty$ from the system dynamics (7). Also, relation (17) gives $\lambda_{\min}(\mathbf{K}_D)\int_0^T \|\mathbf{s}(t)\|^2 dt \le V(0), \forall T > 0$, i.e., $\mathbf{s}(t) \in L_2$, where $\lambda_{\min}(\mathbf{K}_D)$ denotes the minimum eigenvalue of $\mathbf{K}_D$. Applying Barbalat's lemma [11] yields $\lim_{t\to\infty}\mathbf{s}(t) = 0$. However, the control (12) is still discontinuous at $s_i(t) = 0$, and thus is not readily implemented. As a next stage, we make the control action continuous by a standard modification technique which leads to a practically implementable control law.

B. Continuous Adaptive Control Design

A continuous control action can be derived by modifying the velocity error $\mathbf{s}(t)$. First, introduce a new variable $\mathbf{s}_\varepsilon(t)$ by setting

$$\mathbf{s}_\varepsilon = \mathbf{s} - \frac{1}{\sqrt{3}}\mathbf{c}(\mathbf{s}) \tag{18}$$

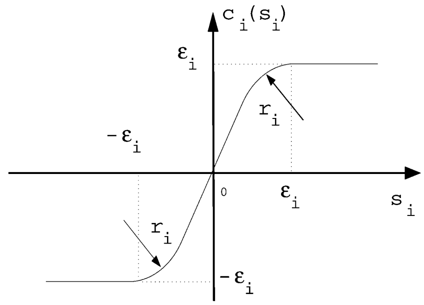

Fig. 1. Smooth function $c_i(s_i)$.

where

$$\mathbf{c}(\mathbf{s}) = [c_1(s_1) \quad \cdots \quad c_n(s_n)]^T$$
$$c_i(s_i) = \begin{cases} b_i + \sqrt{r_i^2 - (s_i - \varepsilon_i)^2}, & \frac{\sqrt{3}-1}{2}\varepsilon_i \le s_i \le \varepsilon_i \\ \sqrt{3}s_i, & |s_i| \le \frac{\sqrt{3}-1}{2}\varepsilon_i \\ -b_i - \sqrt{r_i^2 - (s_i + \varepsilon_i)^2}, & -\varepsilon_i \le s_i \le -\frac{\sqrt{3}-1}{2}\varepsilon_i \\ \varepsilon_i \mathrm{sgn}(s_i), & |s_i| > \varepsilon_i \end{cases} \tag{19}$$

with $r_i = (\sqrt{3}-1)\varepsilon_i, b_i = (2-\sqrt{3})\varepsilon_i$, for $\varepsilon_i > 0 \, \forall i = 1, \ldots, n$.

It is standard to show that such $\mathbf{s}_\varepsilon(t)$ is continuously differentiable in time t (see also Fig. 1). Using Property 2.3, the dynamics of system (1) in terms of the modified "velocity error" $\mathbf{s}_\varepsilon(t)$ is expressed by

$$\mathbf{H}(\mathbf{q})\dot{\mathbf{s}}_\varepsilon + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{s}_\varepsilon = \boldsymbol{\tau} - \mathbf{Y}_\varepsilon \mathbf{a} - \mathbf{f}_N(\mathbf{x}, \boldsymbol{\theta}) \tag{20}$$

where $\mathbf{Y}_\varepsilon = \mathbf{H}(\mathbf{q})(\ddot{\mathbf{q}}_r + (1/\sqrt{3})\dot{\mathbf{c}}(\mathbf{s})) + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})(\dot{\mathbf{q}}_r + (1/\sqrt{3})\mathbf{c}(\mathbf{s})) + \mathbf{g}(\mathbf{q})$.

Now, take the following Lyapunov function

$$V_\varepsilon(t) = \frac{1}{2}\mathbf{s}_\varepsilon^T \mathbf{H}(\mathbf{q})\mathbf{s}_\varepsilon + \frac{1}{2}\left(\tilde{\mathbf{a}}^T \mathbf{\Gamma}_a^{-1}\tilde{\mathbf{a}} + \tilde{\boldsymbol{\beta}}^T \mathbf{\Gamma}_\beta^{-1}\tilde{\boldsymbol{\beta}}\right). \tag{21}$$

Like (11), it is clear that

$$\dot{V}_\epsilon(t) \le \mathbf{s}_\varepsilon^T(\boldsymbol{\tau} - \mathbf{Y}_\varepsilon \mathbf{a}) + \mathbf{s}_\varepsilon^T \mathbf{f}_N(\mathbf{x}, 0) + \mathbf{s}_\varepsilon^T \mathbf{\Phi}(\mathbf{s}_\varepsilon, \mathbf{x})\boldsymbol{\beta} + \dot{\hat{\mathbf{a}}}^T \mathbf{\Gamma}_a^{-1}\tilde{\mathbf{a}} + \dot{\hat{\boldsymbol{\beta}}}^T \mathbf{\Gamma}_\beta^{-1}\tilde{\beta} \tag{22}$$

where let us recall that $\mathbf{\Phi}$ is already defined by formula (10).

Note that whenever $|s_i| \le ((\sqrt{3}-1)/2)\varepsilon_i$, one has

$$s_{i\varepsilon} = 0$$

and for $|s_i| > ((\sqrt{3}-1)/2)\varepsilon_i$

$$\begin{aligned} s_{i\varepsilon}^2 &\le s_{i\varepsilon}s_i \\ \mathrm{sgn}(s_i) &= \mathrm{sgn}(s_{i\varepsilon}). \end{aligned} \tag{23}$$

Hence, introducing the saturated function

$$\mathrm{sat}_{\varepsilon_i}(s_i) = \begin{cases} \frac{s_i}{\frac{\sqrt{3}-1}{2}\varepsilon_i} & \text{when } |s_i| \le \frac{\sqrt{3}-1}{2}\varepsilon_i \\ \mathrm{sgn}(s_i) & \text{when } |s_i| > \frac{\sqrt{3}-1}{2}\varepsilon_i \end{cases} \tag{24}$$

and taking (22) and (23) into account, the following continuous control input:

$$\boldsymbol{\tau} = -\mathbf{K}_D\mathbf{s} + \mathbf{Y}_\varepsilon\hat{\mathbf{a}} - \mathbf{f}_N(\mathbf{x}, 0) - \mathbf{\Phi}_\varepsilon(\mathbf{s}, \mathbf{x})\hat{\boldsymbol{\beta}} \tag{25}$$

with

$$\mathbf{\Phi}_\varepsilon(\mathbf{s}, \mathbf{x}) := \mathrm{diag}\left[\mathrm{sat}_{\varepsilon_1}(s_1)\mathbf{w}_1(x), \ldots, \mathrm{sat}_{\varepsilon_n}(s_n)\mathbf{w}_n(x)\right] \in R^{n\times 2n}$$

together with the update laws

$$\begin{aligned} \dot{\hat{\mathbf{a}}} &= -\Gamma_a \mathbf{Y}^T \mathbf{s}_\varepsilon \\ \dot{\hat{\boldsymbol{\beta}}} &= \Gamma_\beta \mathbf{W}^T(\mathbf{x})|\mathbf{s}_\varepsilon| \\ |\mathbf{s}_\epsilon| &= [|s_{1\epsilon}| \quad |s_{2\epsilon}| \quad \cdots \quad |s_{n\epsilon}|]^T \end{aligned} \tag{26}$$

yield

$$\dot{V}_\varepsilon(t) \le -\mathbf{s}_\varepsilon^T \mathbf{K}_D \mathbf{s}_\varepsilon.$$

Finally, by a similar analysis, as done in Section III-A, the error $\mathbf{s}_\varepsilon(t)$ of the system converges to 0, or equivalently $\lim_{t\to\infty}|s_i| \le ((\sqrt{3}-1)/2)\varepsilon_i, i = 1, \ldots, n$. From relation (8), the tracking error $\tilde{q}_i(t)$ converges to $((\sqrt{3}-1)/(2\lambda_i))\varepsilon_i$ as $t \to \infty$. We are now in a position to sum up our results.

Theorem 1: The adaptive controller defined by (18), (19), and (24)–(26) enables system (1) to asymptotically track a desired trajectory $\mathbf{q}_d(t)$ within a precision of $((\sqrt{3}-1)/(2\lambda_i))\varepsilon_i, i = 1, \ldots, n$.

Remark 1: In the general case where $\boldsymbol{\theta}_i \in R^{p_i}$, it follows in a straightforward manner from Lemma 1 that

$$\begin{aligned} &e(t)\mathbf{g}_i(\mathbf{x}, \boldsymbol{\theta}_i)\mathbf{h}_i(\mathbf{x}, \boldsymbol{\theta}_i) \\ &\quad \le e(t)\mathbf{g}_i(\mathbf{x}, 0)\mathbf{h}_i(\mathbf{x}, 0) + |e(t)| \\ &\quad\quad \times \left\{ L_i(\mathbf{x})\ell_i(\mathbf{x}) \left(\sum_{j=1}^{p_i}|\theta_{ij}|\right)^2 \right. \\ &\quad\quad\quad + [\|\mathbf{h}_i(\mathbf{x}, 0)\|L_i(\mathbf{x}) + \|\mathbf{g}_i(\mathbf{x}, 0)\|\ell_i(\mathbf{x})] \\ &\quad\quad\quad \left. \times \sum_{j=1}^{p_i}|\theta_{ij}| \right\}. \end{aligned}$$

Therefore, with a Lyapunov function defined in (21), where

$$\boldsymbol{\beta}_i = [\beta_{i1}, \beta_{i2}]^T = \left[\left(\sum_{j=1}^{p}|\theta_{ij}|\right)^2, \quad \sum_{j=1}^{p}|\theta_{ij}|\right]^T.$$

Theorem 1 remains valid for $\boldsymbol{\theta}_i \in R_i^p$.

Remark 2: The framework of this brief together with the approach of [13] can provide a construction technique for a broad class of nonlinearly parameterized uncertainties not only in multiplicative form, but also in fractional form and their combinations thereof.

Remark 3: The new variable (18) and the function (19) are properly designed to make the stabilizing control (12) continuous. Of course, there are other appropriate choices other than the variable (18) and the function (19), which also make the stabilizing control (12) continuous, too.

C. 1-D Estimator

In the design of Sections III-A and III-B, the dimensions of estimators are equal to the number of unknown parameters in the system, i.e., $\hat{\mathbf{a}} \in R^{\omega}, \hat{\boldsymbol{\beta}} \in R^{2n}$. Thus, increasing the number of links may result in estimators of excessively large dimension. Tuning updating gains $\boldsymbol{\Gamma}_a, \boldsymbol{\Gamma}_\beta$ for those estimators then becomes a very laborious task. In this section, we show that it is possible to design an adaptive controller for system (1) with simple 1-D estimators $\hat{a}, \hat{\beta}$ independently of the dimensions of the unknown parameters $\mathbf{a}$ and $\boldsymbol{\beta}$.

For that purpose, first consider the term $\mathbf{Ya}$ in (9), where $\mathbf{Y} \in R^{n\times\omega}, \mathbf{a} \in R^{\omega}$. It is clear that

$$\sum_{j=1}^{\omega} Y_{ij} a_j \leq \left(\max_{j=1,\ldots,\omega} |Y_{ij}|\right) \sum_{j=1}^{\omega} |a_j|, \quad i = 1, \ldots, n.$$

Also note from (10) that

$$\mathbf{w}_i(\mathbf{x})\boldsymbol{\beta}_i \leq \max_{j=1,2} |w_{ij}(\mathbf{x})|(|\beta_{i1}| + |\beta_{i2}|), \quad i = 1, \ldots, n.$$

As a result, the inequality (11) can be rewritten as follows:

$$\begin{aligned}\dot{V}_1(t) &\leq \mathbf{s}^T\boldsymbol{\tau} + \mathbf{s}^T\mathbf{f}_N(\mathbf{x}, 0)\\ &\quad + |\mathbf{s}^T| \left(\mathbf{y}_{\max} \sum_{j=1}^{\omega} |a_j| + \mathbf{w}_{\max}(\mathbf{x}) \sum_{i=1}^{n} (|\beta_{i1}| + |\beta_{i2}|)\right)\end{aligned}$$

where

$$\begin{aligned}\mathbf{y}_{\max} &:= [\max_{j=1,\ldots,\omega} |Y_{1j}|, \ldots, \max_{j=1,\ldots,\omega} |Y_{nj}|]^T \in R^n\\ \mathbf{w}_{\max}(\mathbf{x}) &:= [\max_{j=1,2} |w_{1j}(\mathbf{x})|, \ldots, \max_{j=1,2} |w_{nj}(\mathbf{x})|]^T \in R^n.\end{aligned}$$

Note that $\mathbf{y}_{\max}$ is the function whose notations on variables $\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r$ are neglected for simplicity. Therefore, with the definitions

$$\begin{aligned}\boldsymbol{\psi}_{\max}(\mathbf{s}) &:= \mathrm{diag}[\mathrm{sgn}(s_1), \ldots, \mathrm{sgn}(s_n)]\mathbf{y}_{\max} \in R^n\\ \boldsymbol{\phi}_{\max}(\mathbf{s}, \mathbf{x}) &:= \mathrm{diag}[\mathrm{sgn}(s_1), \ldots, \mathrm{sgn}(s_n)]\mathbf{w}_{\max}(\mathbf{x}) \in R^n\end{aligned}$$

the following control input:

$$\begin{aligned}&\boldsymbol{\tau} = -\mathbf{K}_D\mathbf{s} - \mathbf{f}_N(\mathbf{x}, 0) - (\boldsymbol{\psi}_{\max}(\mathbf{s})\hat{a} + \boldsymbol{\phi}_{\max}(\mathbf{s}, \mathbf{x})\hat{\beta})\\ &\dot{\hat{a}} = \gamma_a \mathbf{y}_{\max}^T|\mathbf{s}|, \dot{\hat{\beta}} = \gamma_\beta \mathbf{w}_{\max}^T(\mathbf{x})|\mathbf{s}|\end{aligned} \tag{27}$$

where γ_a and γ_β are arbitrary positive scalars, together with the following Lyapunov function:

$$\begin{aligned}V_{1D}(t) = V_1(t) &+ \frac{1}{2}\gamma_a^{-1} \left(\sum_{j=1}^{\omega} |a_j| - \hat{a}\right)^2\\ &+ \frac{1}{2}\gamma_\beta^{-1} \left(\sum_{i=1}^{n} (|\beta_{i1}| + |\beta_{i2}|) - \hat{\beta}\right)^2\end{aligned} \tag{28}$$

yield

$$\dot{V}(t)_{1D} \leq -\mathbf{s}^T\mathbf{K}_D\mathbf{s}.$$

Therefore, the discontinuous control (27) results in the convergence to 0 of velocity error $\mathbf{s}(t)$, which ensures the convergence to 0 of tracking error $\tilde{\mathbf{q}}(t)$ when $t \to \infty$. As in Section III-B, we can alter the discontinuous control (27) into a continuous one as follows:

$$\begin{aligned}&\boldsymbol{\tau} = -\mathbf{K}_D\mathbf{s} - \mathbf{f}_N(\mathbf{x}, 0) - (\boldsymbol{\psi}_{\max\varepsilon}(\mathbf{s})\hat{a} + \boldsymbol{\phi}_{\max\varepsilon}(\mathbf{s}, \mathbf{x})\hat{\beta})\\ &\dot{\hat{a}} = \gamma_a \mathbf{y}_{\max}^T|\mathbf{s}_\varepsilon|\\ &\dot{\hat{\beta}} = \gamma_\beta \mathbf{w}_{\max}^T(\mathbf{x})|\mathbf{s}_\varepsilon|\end{aligned} \tag{29}$$

where

$$\begin{aligned}\boldsymbol{\psi}_{\max\varepsilon}(\mathbf{s}) &:= \mathrm{diag}\left[\mathrm{sat}_{\varepsilon_1}(s_1), \ldots, \mathrm{sat}_{\varepsilon_n}(s_n)\right]\mathbf{y}_{\max} \in R^n\\ \boldsymbol{\phi}_{\max\varepsilon}(\mathbf{s}, \mathbf{x}) &:= \mathrm{diag}\left[\mathrm{sat}_{\varepsilon_1}(s_1), \ldots, \mathrm{sat}_{\varepsilon_n}(s_n)\right]\mathbf{w}_{\max}(\mathbf{x}) \in R^n.\end{aligned}$$

Then the continuous control (29) ensures the convergence to $((\sqrt{3} - 1)/(2\lambda_i))\varepsilon_i, i = 1, \ldots, n$ of the tracking error $\tilde{\mathbf{q}}(t)$ as $t \to \infty$.

IV. Nonlinear Friction Compensation

In this section, we examine how effectively our designed adaptive controllers can compensate for the frictional forces in joints of robot manipulators.

A. Friction Model and Friction Compensators

Frictional forces in system (1) can be described in different ways. Here, we consider the well-known Amstrong–Helouvry model [1]. For joint i, the frictional force is described as

$$\begin{aligned}f_i = F_{ci}\mathrm{sgn}(\dot{q}_i) &\left[1 - \exp\left(-\frac{\dot{q}_i^2}{\upsilon_{si}^2}\right)\right]\\ &+ F_{si}\mathrm{sgn}(\dot{q}_i) \exp\left(-\frac{\dot{q}_i^2}{\upsilon_{si}^2}\right) + F_{\upsilon i}\dot{q}_i\end{aligned} \tag{30}$$

where F_{ci}, F_{si}, F_{vi} are coefficients characterizing the Coulomb friction, static friction, and viscous friction, respectively, and υ_{si} is the Stribeck parameter. Note that the friction term (30) can be decomposed into a linear part f_{Li} and a nonlinear part f_{Ni} as

$$f_i = f_{Li} + f_{Ni} \tag{31}$$

where

$$f_{Li} = F_{ci}\mathrm{sgn}(\dot{q}_i) + F_{vi}\dot{q}_i = \mathbf{z}_i\boldsymbol{\alpha}_i \tag{32}$$

with $\boldsymbol{\alpha}_i = [F_{ci} \ \ F_{vi}]^T$, $\mathbf{z}_i = [\mathrm{sgn}(\dot{q}_i) \ \ \dot{q}_i]$, and

$$f_{Ni} = (F_{si} - F_{ci})\mathrm{sgn}(\dot{q}_i)\exp\left(-\frac{\dot{q}_i^2}{v_{si}^2}\right). \tag{33}$$

Practically, the frictional coefficients are not exactly known. In such a case, the frictional force f_{Li} can be compensated by a traditional adaptive control for LP. However, the situation becomes nontrivial when there are unknown parameters appearing nonlinearly in the model of f_{Ni}.

The NP friction term of joint i, f_{Ni}, can be expressed in the form (2) with

$$f_{Ni} = g_i(\dot{q}_i, \boldsymbol{\theta}_i)h_i(\dot{q}_i, \boldsymbol{\theta}_i) \tag{34}$$

where

$$\boldsymbol{\theta}_i = \left[(F_{si} - F_{ci}) \quad \frac{1}{v_{si}^2}\right]^T = [\theta_{i1} \quad \theta_{i2}]^T$$
$$g_i(\dot{q}_i, \theta_i) = [1 \quad 0]\theta_i, h_i(\dot{q}_i, \theta_i) = \mathrm{sgn}(\dot{q}_i)\exp\left(-\dot{q}_i^2\theta_{i2}\right).$$

Clearly, g_i and h_i are Lipschitzian in θ_i with Lipschitzian coefficients $l_i(\dot{q}) = \dot{q}_i^2, L_i(\dot{q}) = 1$. Also, we have $g_i(\dot{q}_i, 0) = 0, h_i(\dot{q}_i, 0) = 1$. Therefore, by Theorem 1, the following adaptive controller enables the system (1), (30), and (34) to asymptotically track a desired trajectory $q_{di}(t)$ within a precision of $((\sqrt{3}-1)/(2\lambda_i))\varepsilon_i, i = 1, \ldots, n$:

$$\begin{aligned}\boldsymbol{\tau} &= -\mathbf{K}_D\mathbf{s} + \mathbf{Y}_\varepsilon\hat{\mathbf{a}} + \mathbf{Z}\hat{\boldsymbol{\alpha}} - \boldsymbol{\Phi}_\varepsilon(\mathbf{s}, \mathbf{x})\hat{\boldsymbol{\beta}} \\ \dot{\hat{\mathbf{a}}} &= -\boldsymbol{\Gamma}_a\mathbf{Y}^T\mathbf{s}_\varepsilon, \dot{\hat{\boldsymbol{\alpha}}} = -\boldsymbol{\Gamma}_\alpha\mathbf{Z}^T\mathbf{s}_\varepsilon, \dot{\hat{\boldsymbol{\beta}}} = \boldsymbol{\Gamma}_\beta\mathbf{W}^T(\mathbf{x})|\mathbf{s}_\varepsilon|\end{aligned} \tag{35}$$

where

$$\begin{aligned}\mathbf{Z} &= \mathrm{diag}[\mathbf{z}_1(q_1), \ldots, \mathbf{z}_n(q_n)] \in R^{n\times 2n} \\ \mathbf{w}_i(\dot{\mathbf{q}}) &= \left[\dot{q}_i^2, 1\right].\end{aligned} \tag{36}$$

Note that with the control (35), the term $\mathbf{Z}\hat{\boldsymbol{\alpha}}$ compensates for the LP frictions f_{Li}.

B. Simulations

A prototype of a planar 2DOF robot manipulator is built to assess the validity of the proposed methods (see Fig. 2). The dynamic model of the manipulator and its linearized dynamics parameter are given in Appendix B.

The manipulator model is characterized by a real parameter $\mathbf{a}$, which is identified by a standard technique (See Table III in Appendix B). The parameters of friction model (30) are chosen such that the effect of the NP frictions f_{Ni} are significant, i.e.,

$$\begin{aligned}F_{ci} &= 0.49 \\ Fsi &= 3.5 \\ F_{vi} &= 0.15 \\ v_{si} &= 0.189 \\ \forall i &= 1, 2.\end{aligned}$$

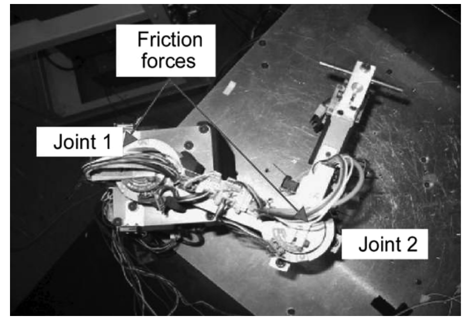

Fig. 2. Prototype of robot manipulator.

TABLE I
PARAMETERS OF THE CONTROLLERS FOR SIMULATIONS

$\mathbf{K}_D$	$\boldsymbol{\Lambda}$	$\boldsymbol{\Gamma}_a$	$\boldsymbol{\Gamma}_\alpha$
$10\mathbf{I}(2,2)$	$5\mathbf{I}(2,2)$	diag(5,5,5)	diag(3,3,3,3)

In order to focus on the compensation of nonlinearly parameterized frictions, we have selected the objective of low-velocity tracking. The manipulator must track the desired trajectory $q_{d1}(t) = (\pi/6)(1 - \cos(t)), q_{d2}(t) = (\pi/4)(1 - \cos(t))$. Clearly, the selected trajectory contains various zero velocity crossings.

For comparison, we use the following two different controllers to accomplish the tracking task.

- A traditional adaptive control based on the LP structure to compensate for uncertainty in dynamic parameter $\mathbf{a}$ of the manipulator links and the linearly parameterized frictions f_{Li} (32) in joints of motors

$$\begin{aligned}\boldsymbol{\tau} &= -\mathbf{K}_D\mathbf{s} + \mathbf{Y}\hat{\mathbf{a}} + \mathbf{Z}\hat{\boldsymbol{\alpha}} \\ \dot{\hat{\mathbf{a}}} &= -\boldsymbol{\Gamma}_a\mathbf{Y}^T\mathbf{s}, \quad \dot{\hat{\boldsymbol{\alpha}}} = -\boldsymbol{\Gamma}_\alpha\mathbf{Z}^T\mathbf{s}.\end{aligned} \tag{37}$$

  The gains of the controller are chosen as in Table I, $\hat{\mathbf{a}} \in R^3, \hat{\boldsymbol{\alpha}} \in R^4$.
- Our proposed controller (35) with the same control parameters for LP uncertainties. Additionally, $\boldsymbol{\Gamma}_\beta = \mathrm{diag}(50, 50, 50, 50), \epsilon = 0.05$ for NP friction compensation, $\hat{\boldsymbol{\beta}} \in R^4$.

Both controllers start without any prior information of dynamic and frictional parameters, i.e., $\hat{\mathbf{a}}(0) = 0, \hat{\boldsymbol{\alpha}}(0) = 0, \hat{\boldsymbol{\beta}}(0) = 0$.

Tradition LP Adaptive Control Versus Proposed Control: It can be seen that the position error is much smaller with the proposed control (see Fig. 3), especially at points where manipulator velocities cross the value of zero. Indeed, the position error of joint 1 decreases about 20 times. The position tracking of joint 2 is improved in the sense that our proposed control obtains a same level of position error as the one of LP, but the bound of control input is reduced about three times. This means that the nonlinearly parameterized frictions are effectively compensated by our method.

1-D-Estimators: The performances of the controller with 1-D-estimators (29) is shown in Fig. 4. One estimate is designed for the manipulator dynamics $\mathbf{a} \in R^3$, one is for the LP friction parameters $\boldsymbol{\alpha} \in R^4$, and one is for the NP friction parameters $\boldsymbol{\beta} \in R^4$. Thus, by using 1-D-estimators,

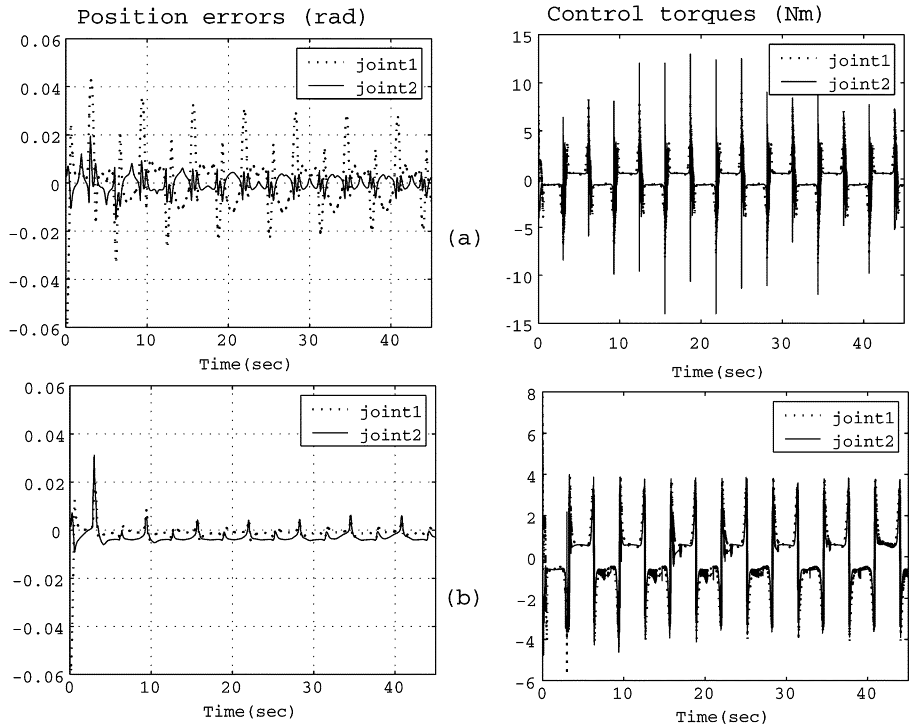

Fig. 3. Simulation results: (left) Tracking errors of joints and (right) characteristics of control inputs. (a) Traditional LP adaptive controller (37). (b) Proposed controller (35).

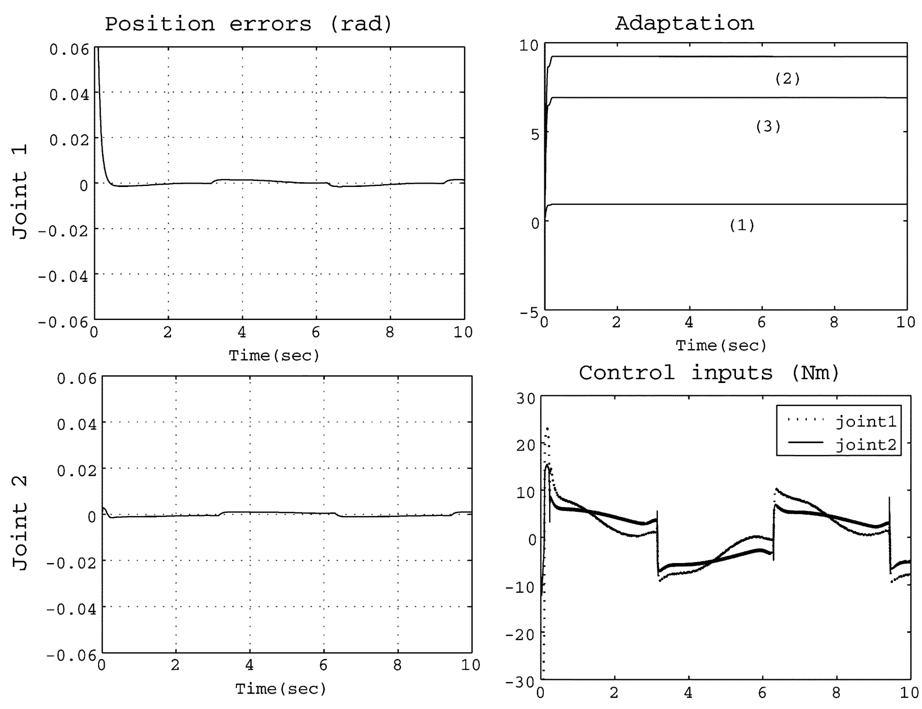

Fig. 4. Simulation results for proposed 1-D-estimators (29): (left) Tracking errors of joints (right) the adaptation of the estimates and characteristics of control inputs. (1) $\hat{\mathbf{a}}$; (2) $\hat{\boldsymbol{\alpha}}$; (3) $\boldsymbol{\beta}$.

the estimates dimension reduces from 11 to 3. The resulting controller benefits not only from a simpler tuning scheme, but also from a minimum amount of online calculation since the regressor matrices $\mathbf{Y}, \mathbf{W}$ reduce to the vectors $\mathbf{y}_{\max}, \mathbf{w}_{\max}$ in this case. Indeed, under the current simulation environment (WindowsXP/MATLAB Simulink), controller (29) requires a computation load 0.7 time less than the one of controller (35) and only 1.2 times bigger than the one of tradition LP adaptive control (37). Also, it can be seen in Fig. 4 that these advantages result in a faster convergence (just few

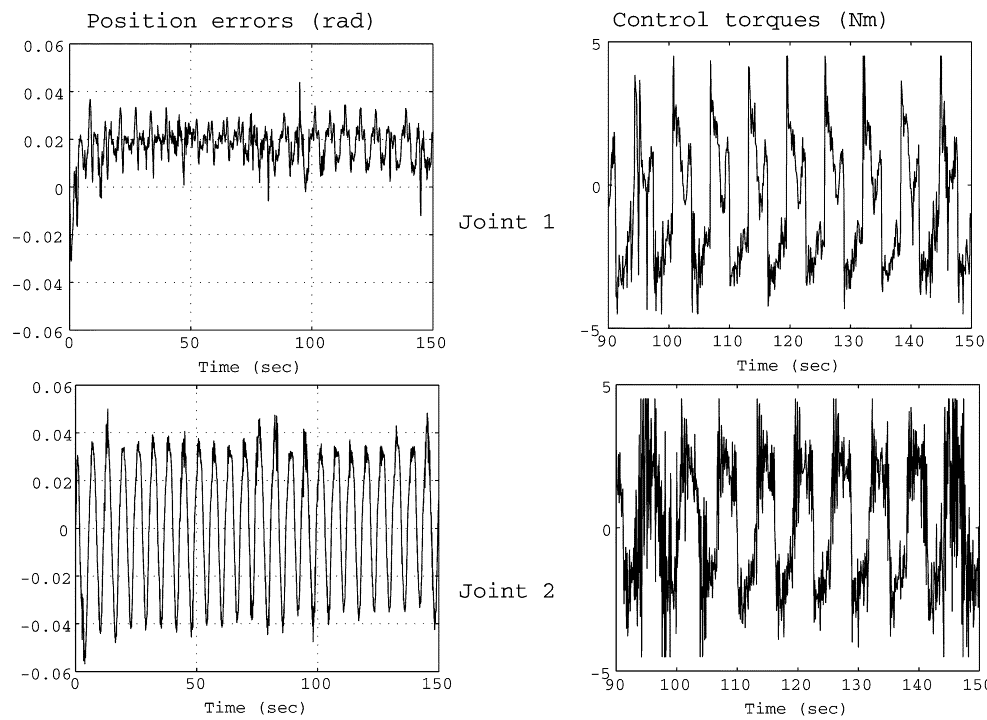

Fig. 5. Experimental results for traditional LP adaptive controller (37): (left) Tracking errors of joints and (right) characteristics of control inputs.

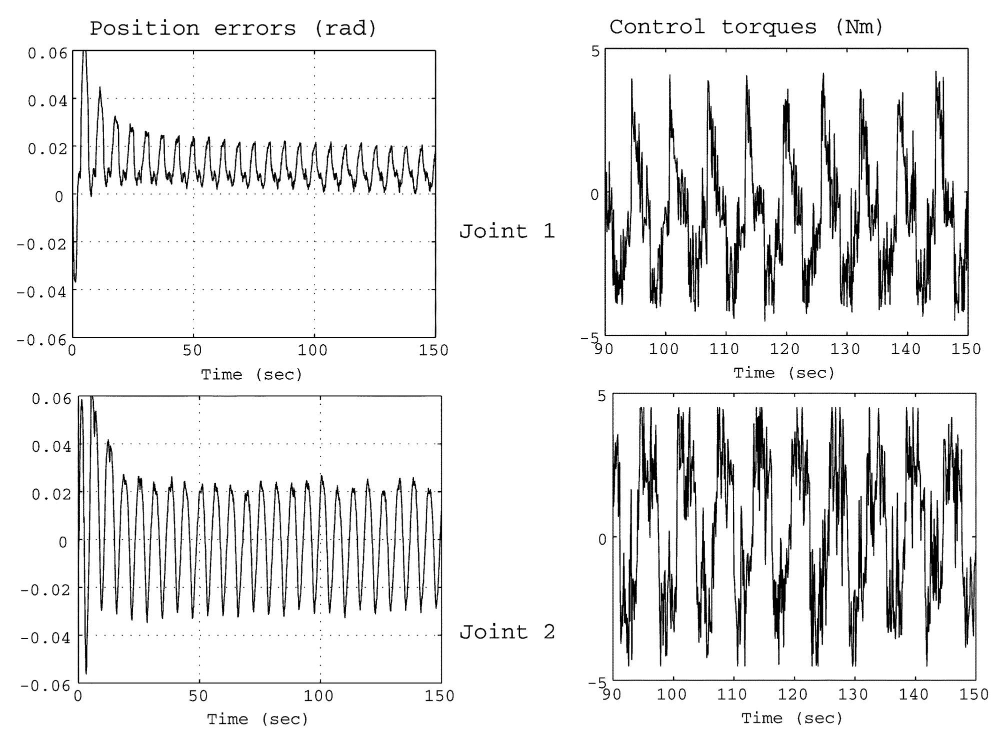

Fig. 6. Experimental results for proposed controller (35): (left) Tracking errors of joints and (right) characteristics of control inputs.

instants after the initial time) of the tracking errors to the designed value (0.0035 rad in this simulation). Note that the estimates converge to constant values since the adaptation mechanism in controller (29) becomes standstill whenever the tracking errors become less than the design value. However, it is worth noting that the maximum value of control inputs of controller (29), which is required only at the adaptation process of the estimates, is about six times bigger than the one of controller (35). It can be learned from the simulation result that controller (29) can effectively compensate the NP uncertainties in the system provided that there is no limitation to the control inputs. Therefore, controller (35) can be a good choice for practical applications whose the power of actuators are constrained.

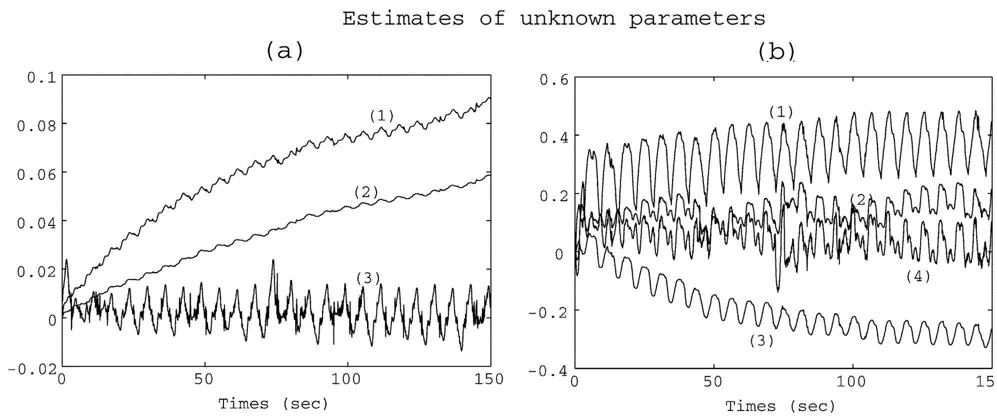

Fig. 7. Experimental results: Estimates of unknown parameters with traditional LP adaptive controller (37). (a) Estimate $\hat{\mathbf{a}}$: (1)—$\hat{a}_1$, (2)—$\hat{a}_2$, (3)—$\hat{a}_3$. (b) Estimate $\hat{\boldsymbol{\alpha}}$: (1) $\hat{\alpha}_1$; (2) $\hat{\alpha}_2$; (3) $\hat{\alpha}_3$; (4) $\hat{\alpha}_4$.

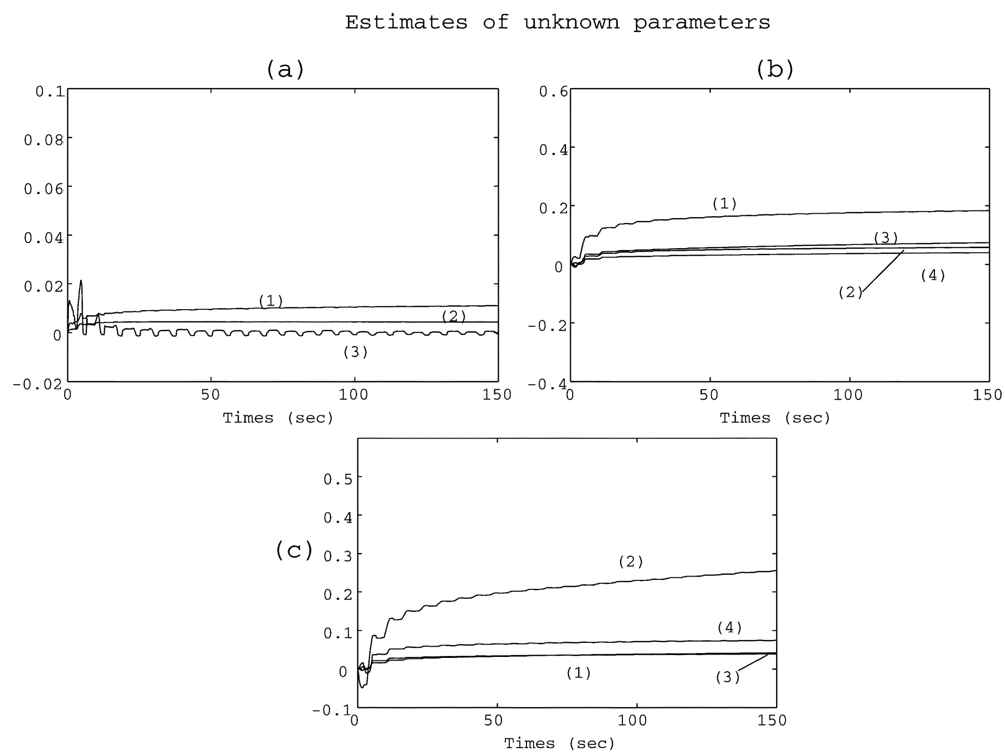

Fig. 8. Experimental results: Estimates of unknown parameters with proposed controller (35). (a) Estimate $\hat{\mathbf{a}}$: (1) $\hat{\alpha}_1$; (2) $\hat{\alpha}_2$; (3) $\hat{\alpha}_3$. (b) Estimate $\hat{\boldsymbol{\alpha}}$: (1) $\hat{\alpha}_1$; (2) $\hat{\alpha}_2$; (3) $\hat{\alpha}_3$; (4) $\hat{\alpha}_4$. (c) Estimate $\hat{\boldsymbol{\beta}}$: (1) $\hat{\beta}_1$; (2) $\hat{\beta}_2$; (3) $\hat{\beta}_3$; (4) $\hat{\beta}_4$.

C. Experiments

All joints of the manipulator are driven by YASKAWA dc motors UGRMEM-02SA2. The range of motor power is $[-5, 5]$ Nm. The joint angles are detected by potentiometers (350°, ± 0.5). Control input signals are sent to each dc motor via a METRONIX amplifier (± 35 V, ± 3 A). The joint velocities are also calculated from the derivation of joint positions with low-pass filters. Designed controller is implemented on ADSP324-00A, 32-bit DSP board with 50 MHz CPU clock. I/O interface is ADSP32X-03/53, 12 bit A/D, D/A card. The DSP and the interface card are mounted on Windows98-based PC. The sampling time is 2 ms.

Here again, the performances of controller (37) and the proposed control (35) are compared. The gains of the controllers are chosen as in Table II. The additional control parameters for NP friction compensation with (35) are $\mathbf{\Gamma}_\beta = \mathrm{diag}(1, 1, 1, 1), \epsilon = 0.1$.

Fig. 5 depicts the performances of LP adaptive controller (37). The fact that the trajectory tracking error of joint 2 become about twice smaller as shown by Fig. 6 highlights how effectively the NP frictions are compensated by the proposed controller. The estimates of unknown parameters with adaptation mechanisms in LP adaptive controller (37) and proposed controller (35) are shown by Figs. 7 and 8, respectively. Since the adaptation mechanism of LP adaptive controller (37) can not compensate for the NP friction terms, its estimates can not converge to any values able to make the trajectory tracking errors converge to 0. For the proposed controller, a better convergence

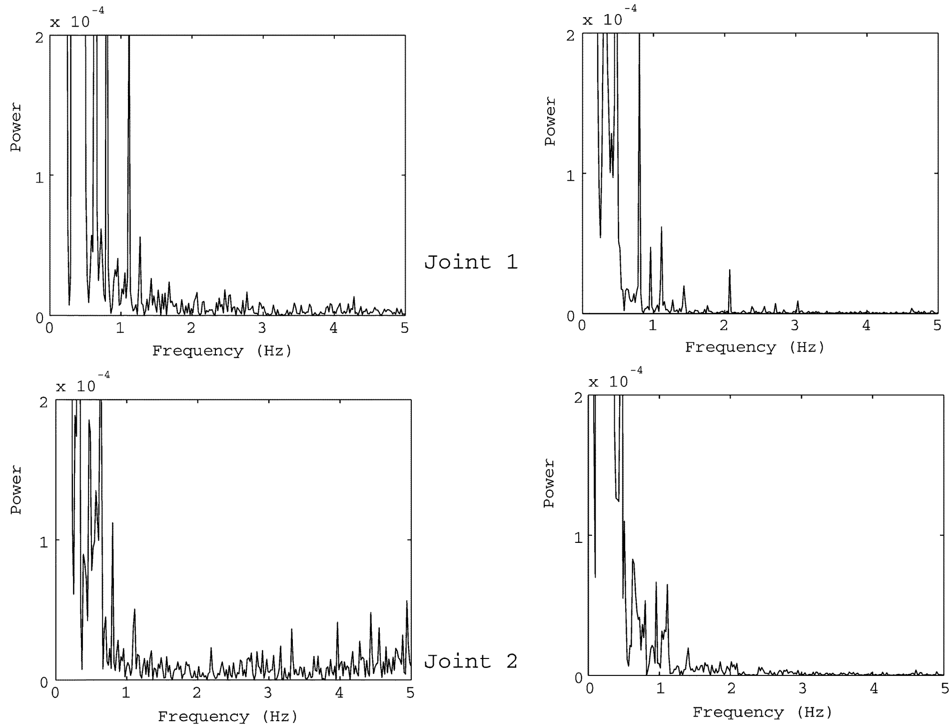

Fig. 9. Experimental results: FFT of trajectory tracking errors for (left) traditional LP adaptive controller (37) and (right) proposed controller (35).

TABLE II
PARAMETERS OF THE CONTROLLERS FOR EXPERIMENTS

$\mathbf{K}_D$	$\mathbf{\Lambda}$	$\mathbf{\Gamma}_a$	$\mathbf{\Gamma}_\alpha$
$3\mathbf{I}(2,2)$	$3\mathbf{I}(2,2)$	diag(.05,.05,.05)	diag(2.5,5,2.5,5)

of the estimates can be observed. That the motion of the manipulator has lower frequencies in case of the proposed control (see Fig. 9) shows its more robustness in face of noisy inputs. These results can be obtained because the NP frictions are compensated effectively.

Remark 4: The non-zero mean value of the tracking errors is caused by the presence of unknown disturbances or unmodeled dynamics. These dynamics, therefore, may cause the drift in the parameter estimates. Other well-known techniques using prior information or update law modification [10] may be additionally applied for robustness improvement of the adaptive schemes.

Remark 5: The experiment using the controller with 1-D-estimators is not reported here because the power required for the execution of the control inputs was higher than the maximum power of the robot actuators.

V. CONCLUSION

We have developed a new adaptive control framework which applies to any nonlinearly parameterized system which satisfies a general Lipschitzian property. This allows us to extend the scope of adaptive control to handle very general control problems for robot manipulators since Lipschitzian parameterizations include as special cases convex/concave and smooth parameterizations. As byproducts, the approach permits also to treat uncertainties in fractional form, multiplicative form, and their combinations thereof. When the structure of adaptive control for NP is exploited to redesign the traditional adaptive control for LP, we have shown how an adaptive control with low-dimensional estimators (1-D-estimators) can be designed. This means system designers will have more freedom to design their tuning schemes by balancing the dimension of the design estimators and the power required by system control inputs. This brief opens an approach to implement adaptive controls for robot manipulators as well as facilitates the tuning of practical control systems.

APPENDIX

A. Proof of Lemma 1

Useful properties for adaptation of NP to Lipschitzian functions appearing in multiplicative form can be found in [13]. Here, we give a brief proof for Lemma 1. Since

$$\begin{aligned} e(t)[\mathbf{g}(\mathbf{x},\boldsymbol{\theta})\mathbf{h}(\mathbf{x},\theta) - \mathbf{g}(\mathbf{x},0)\mathbf{h}(\mathbf{x},0)] \\ \le |e(t)||\mathbf{g}(\mathbf{x},\boldsymbol{\theta})\mathbf{h}(\mathbf{x},\boldsymbol{\theta}) - \mathbf{g}(\mathbf{x},0)\mathbf{h}(\mathbf{x},0)| \end{aligned}$$

it is sufficient to prove that

$$\begin{aligned} &|\mathbf{g}(\mathbf{x},\boldsymbol{\theta})\mathbf{h}(\mathbf{x},\boldsymbol{\theta}) - \mathbf{g}(\mathbf{x},0)\mathbf{h}(\mathbf{x},0)| \\ &\quad \le \Bigg\{ L(\mathbf{x})\ell(\mathbf{x}) \left(\sum_{j=1}^{p}\theta_j\right)^2 \\ &\qquad + [\|\mathbf{h}(\mathbf{x},0)\|\, L(\mathbf{x}) + \|\mathbf{g}(\mathbf{x},0)\|\, \ell(\mathbf{x})] \sum_{j=1}^{p}\theta_j \Bigg\} \quad (38) \end{aligned}$$

TABLE III
PARAMETERS OF THE 2DOF MANIPULATOR

a_1	a_2	kr_1	kr_2	$m_{l_2}a_1l_2$	d_{11}^*	d_{22}
0.15	0.15	1	1	0.0043	0.2602	0.0188

where $L(\mathbf{x}), \ell(\mathbf{x})$ are defined in (4) and note that the subscripts i is neglected for simplicity.

Actually

$$\begin{aligned}|\mathbf{g}(\mathbf{x},\boldsymbol{\theta})\mathbf{h}(\mathbf{x},\boldsymbol{\theta}) - \mathbf{g}(\mathbf{x},0)\mathbf{h}(\mathbf{x},0)| &\\ = |[\mathbf{g}(\mathbf{x},\boldsymbol{\theta}) - \mathbf{g}(\mathbf{x},0)]\mathbf{h}(\mathbf{x},\boldsymbol{\theta}) + \mathbf{g}(\mathbf{x},0)[\mathbf{h}(\mathbf{x},\boldsymbol{\theta}) - \mathbf{h}(\mathbf{x},0)]| &\\ \leq \|\mathbf{g}(\mathbf{x},\boldsymbol{\theta}) - \mathbf{g}(\mathbf{x},0)\|\|\mathbf{h}(\mathbf{x},\boldsymbol{\theta})\| &\\ + \|\mathbf{g}(\mathbf{x},0)\|\|\mathbf{h}(\mathbf{x},\boldsymbol{\theta}) - \mathbf{h}(\mathbf{x},0)\| &\\ \leq \left(\sum_{j=1}^{p} L_j(\mathbf{x})\theta_j\right)\|\mathbf{h}(\mathbf{x},\boldsymbol{\theta})\| + \|\mathbf{g}(\mathbf{x},0)\|\left(\sum_{j=1}^{p} l_j(x)\theta_j\right) &\\ \leq L(\mathbf{x})\left(\sum_{j=1}^{p}\theta_j\right)\|\mathbf{h}(\mathbf{x},\boldsymbol{\theta})\| + \|\mathbf{g}(\mathbf{x},0)\| l(\mathbf{x})\left(\sum_{j=1}^{p}\theta_j\right) &\end{aligned} \tag{39}$$

and

$$\begin{aligned}\|\mathbf{h}(\mathbf{x},\boldsymbol{\theta})\| &\leq \|\mathbf{h}(\mathbf{x},\boldsymbol{\theta}) - \mathbf{h}(\mathbf{x},0)\| + \|\mathbf{h}(\mathbf{x},0)\| \\ &\leq l(\mathbf{x})\left(\sum_{j=1}^{p}\theta_j\right) + \|\mathbf{h}(\mathbf{x},0)\|.\end{aligned} \tag{40}$$

leads to (38).

B. Model and Parameters of the Manipulator

The equation of motion in joint space for a planar 2DOF manipulator is

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} = \boldsymbol{\tau}$$

or

$$\begin{bmatrix} b_{11}(q_2) & b_{12}(q_2) \\ b_{21}(q_2) & b_{22}\end{bmatrix}\begin{bmatrix}\ddot{q}_1 \\ \ddot{q}_2\end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22}\end{bmatrix}\begin{bmatrix}\dot{q}_1 \\ \dot{q}_2\end{bmatrix} = \boldsymbol{\tau} \tag{41}$$

where

$$\begin{aligned} b_{11} &= \underbrace{I_{l_1} + m_{l_1}l_1^2 + kr_1^2 I_{m_1} + m_{l_2}a_1^2 + m_{m_2}a_1^2}_{d_{11}^*} \\ &\quad + I_{l_2} + I_{m_2} + m_{l_2}l_2^2 + 2m_{l_2}a_1l_2\cos(q_2) \\ b_{12} &= b_{21} = \underbrace{I_{l_2} + kr_2 I_{m_2} + m_{l_2}l_2^2}_{d_{12}} + m_{l_2}a_1l_2\cos(q_2) \\ b_{22} &= \underbrace{I_{l_2} + m_{l_2}l_2^2 + kr_2^2 I_{m_2}}_{d_{22}} \\ h &= -m_{l_2}a_1l_2\sin(q_2) \\ c_{11} &= h\dot{q}_2 \\ c_{12} &= h(\dot{q}_1 + \dot{q}_2) \\ c_{21} &= -h\dot{q}_1 \\ c_{22} &= 0. \end{aligned}$$

m_{li}, m_{mi} are the masses of link i and motor i, respectively. I_{li}, I_{mi} are the moment of inertia relative to the center of mass of link i and the moment of inertia of motor i. l_i is the distance from the center of the mass of link i to the joint axis. a_i is the length of link i. k_{ri} is the gear reduction ratio of motor i.

A constant vector $\mathbf{a} \in R^3$ of dynamic parameters can be defined as follows:

$$\mathbf{a} = [m_{l_2}a_1l_2 \quad d_{11}^* \quad d_{22}]^T.$$

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