A New Scheme on Robust Observer-Based Control Design for Interconnected Systems With Application to an Industrial Utility Boiler

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***Abstract*—This paper presents a new design algorithm for the decentralized output feedback control problem of large-scale interconnected systems. Each subsystem is composed of a linear (possibly unstable) time-invariant part and an uncertain additive nonlinearity which is a discontinuous function of time and state of the overall system. The nonlinear function is assumed to be bounded by a quadratic inequality, and a decentralized estimated state feedback controller and a decentralized observer are designed for each subsystem, based on linear matrix inequalities. Sufficient conditions for the synthesis of feedback action are provided, under which the proposed controllers and observers can achieve robust stabilization of the overall large-scale system. An attractive feature of the proposed scheme is that it guarantees *connective* stability of the overall system and requires no intersubsystem communication. The controller design is evaluated on a natural circulation drum boiler, where the nonlinear model describes the key dynamical properties of the drum, the risers, the downcomers, and the turbine-generator unit. The linearized system has two poles at origin, one associated with water dynamics and the other with generator dynamics. Simulation results are presented that show the effectiveness of the proposed control against instabilities following sudden load variations. The control is also effective for steady-state operation.**

***Index Terms*—Distributed control, industrial boiler systems, linear matrix inequalities (LMIs), nonlinear systems, observer-based control, robustness.**

I. Introduction

During the last few decades, many researchers in the field of large-scale interconnected systems are devoted to decentralized robust control strategies [1]–[7]. An important motivation for the design of decentralized schemes is that the interconnected systems can be decomposed into low-order subsystems. Therefore, the design procedure gets simplified and the overall computational effort can be shared by all of the subsystem controllers. Also, in diverse fields like power systems, robotics, and space structures, interconnected systems do not live in one piece during operation. In many cases, the subsystems are disconnected and again connected in an unpredictable way to perform programmed tasks [4]. Under such structural reconfigurations, what is required is a control strategy that can guarantee *connective* stability of the overall system [2], [4] or can achieve a desired robust *performance* in the presence of uncertain interconnections [3], [8].

The concept of decentralized control is also interesting because it leads to a reduction in communication overhead. It has been long recognized that the attractive property of decentralized *information structure* disappears when the states of subsystems are not available at the local level. Hence, there are strong research efforts to build decentralized feedback controllers based on measurements only [9], [10] or design decentralized observers to estimate the state of individual subsystems that can be used for state feedback control [2], [11], [12]. However, these results are based on the fact that, in order to utilize the separation principle in the design of individual observers, it is necessary that all observers exchange their computed state estimates. The design is therefore unattractive and can be prone to failure in communication links, delays and cost of inter-subsystem communications. Moreover, as most of the ideas are mainly devoted to linear systems, the controller may not guarantee a satisfactory performance over a wide range of operating conditions.

The last decade has seen a number of new developments in the design of observer based control schemes for special class of nonlinear systems [4], [6]–[8], [13]. It is well known that, as the separation principle may not be applicable to nonlinear systems [14], if the true state is replaced by the estimate given by an observer, then exponential stability of the observer does not, in general, guarantee closed-loop stability. Considering the challenge of designing decentralized observer based controllers, [4] proposed a method of *autonomous* decentralized control design, which requires no communications between the subsystems and, at the same time, guarantees *connective* stability of the overall closed-loop system. The class of nonlinear system covered by this work is also *fairly* general, and the design utilizes the multifaceted tools of linear matrix inequalities (LMI) [15]. However, for a solution to exist, this formulation also requires, for each subsystem, that the number of control inputs must be equal to the dimension of the state, which is very restrictive. To resolve this issue, [6] presented some sufficient conditions of observer-based control design. It utilizes the concept of distance to uncontrollability (unobservability) of a pair of matrices $(A, B)((C, A))$ [16], [17] and is based on the existence of symmetric positive definite solutions to algebraic Riccati equations (AREs). However, it does not maximize the interconnection bounds, which is very important in practical scenarios like power systems and robotics. It also requires selection of many additional design parameters for each subsystem, which has to be done on a trial-and-error basis, and so it may not be very user friendly to engineers. In [7], a sequential two-step design procedure has been introduced to solve a nonconvex optimization problem for the nonlinear system in [4] and [6]. It is interesting that it provides a way to maximize the interconnection bounds; however, the optimization problem cannot give a global minima because the set of solutions are not convex.

In this paper, we propose a new design algorithm for decentralized observer-based controllers. We show that the restrictiveness of the design in [4], [6], and [7] can be resolved by

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providing additional degrees of freedom in the observer design, and the existence of decentralized stabilizing controllers and observers depend on the feasibility of solving an optimization problem in the LMI framework. It is important to note that the algorithm provides only sufficient conditions for quadratic stability. Nevertheless, LMI-based design is always preferable because of its simplicity, its ineffable grace of design, and its ability to capture a wide panorama of control problems [18] and from a numerical perspective, as it not only computes the controller and observer parameters by guaranteeing exponential stabilization, but it also maximizes the interconnection bounds. It is useful to learn that the proposed LMI formulation recognizes the *matching conditions* by returning design parameters for any prescribed bound on the interconnection terms. The control law also guarantees connective stability of the overall system.

To show that the approach is also practically relevant, the controller design is evaluated on the nonlinear model of a power unit which describes the key dynamical properties of drum, risers, downcomers, and the turbine-generator set. The model of [19] is extended here to incorporate the dynamics of governor, turbine, and generator. Thus, interconnection terms between the steam generating unit and the electricity generating unit are identified. This helps us to move the pole associated with the pressure dynamics to the left half plane. Throughout the work, a nonlinear simulation package of boilers called SYNSIM is used, which incorporates the nonlinearities encountered in the true plant. It is developed by Syncrude Canada Limited with the purpose of simulating upset conditions that occurs at irregular intervals as well as a general purpose tool for stability analysis. At present, it is extensively used by the power plant engineers, because the matching between measurement from the true plant outputs and the predictions by SYNSIM is very good. Our experiments with SYNSIM reveals that the overall system is highly *interacting* and *nonlinear*. In addition to it, linearization of the developed nonlinear model gives two poles at the origin: one associated with the water dynamics, and the other with the generator dynamics. Considering these issues, we decomposed the overall system into two subsystems: 1) drum-boiler and 2) governor, turbine, and generator unit. Robust decentralized observer-based controllers are then designed, which can maintain stability in the presence of sudden load variations. The stabilizing effect of controllers are tested for different initial conditions and load changes, and simulation results are provided to show the effectiveness of the proposed approach.

The remainder of this paper is organized as follows. In Section II, the problem we deal with is precisely stated, and decentralized observer-based controllers are designed: sufficient conditions are provided involving LMI feasibility problems. Application of the proposed technique to a power unit is given in Section III with particular attention devoted to modeling and controller design. Finally, Section IV offers some concluding remarks.

II. Decentralized Observer-Based Controllers in the LMI Framework

Consider a nonlinear process of the form [5], [10], [20]

$$\dot{x}(t) = Ax(t) + Bu(t) + Gh_r(t,x), \quad y(t) = Cx(t) \qquad (1)$$

where $x(t) \in \Re^n$ is the state of the system, $u(t) \in \Re^m$ is the input vector, and $y(t) \in \Re^p$ is the output vector. A, B, C, and G are constant $n \times n$, $n \times m$, $p \times n$, and $n \times n$ matrices, respectively. The term $h_r : \Re^{n+1} \longrightarrow \Re^n$ is a piecewise continuous nonlinear function in both arguments t and x, satisfying $h_r(t,0) = 0$. Assume that (A,B) is stabilizable, (C,A) is detectable, and the uncertain term $h_r(t,x)$ is bounded by an inequality [5], [10], [20]

$$h_r^T(t,x)h_r(t,x) \le \alpha^2 x^T H_r^T H_r x \qquad (2)$$

where H_r is a known constant matrix and $\alpha > 0$ is a scalar parameter which can be termed as a degree of robustness. Maximization of α leads to increased robustness of the closed-loop system against uncertain nonlinear perturbations. Now, consider the following open-loop dynamic observer:

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + \hat{u}\\ \hat{y}(t) &= C\hat{x}(t)\end{aligned} \qquad (3)$$

where $\hat{x} \in \Re^n$ is the estimated state vector and $\hat{u} \in \Re^n$ is an additional observer input that can be used to achieve closed-loop estimation. In our approach, a dynamic linear feedback of the following form is used:

$$\begin{aligned}\dot{x}_l &= A_l x_l + B_l C(x - \hat{x})\\ \hat{u} &= C_l x_l + D_l C(x - \hat{x}).\end{aligned}$$

With a static state feedback controller $u = K\hat{x}$, the closed-loop system takes the form

$$\begin{aligned}\dot{z}(t) &= \begin{bmatrix} A + BK & D_l C & C_l \\ 0 & A - D_l C & -C_l \\ 0 & B_l C & A_l \end{bmatrix} z(t) + \begin{bmatrix} 0 \\ G \\ 0 \end{bmatrix} w(t,z)\\ &= \hat{A} z(t) + G_r w(t,z)\end{aligned} \qquad (4)$$

where $z(t) = [\hat{x}^T(t)\ e^T(t)\ x_l^T(t)]^T$, the state estimation error $e = (x - \hat{x})$, and the nonlinear function $h_r(t,x)$ in terms of z is represented by $w(t,z)$, which satisfies the following quadratic bound:

$$\begin{aligned}w^T(t,z)w(t,z) &\le \alpha^2 z^T \begin{bmatrix} H_r^T H_r & H_r^T H_r & 0 \\ H_r^T H_r & H_r^T H_r & 0 \\ 0 & 0 & 0 \end{bmatrix} z\\ &= \alpha^2 z^T(t) H^T H z(t).\end{aligned} \qquad (5)$$

For power systems, vehicle platooning, and other interconnected systems, it is useful to develop results for models that include decentralized observer-based controllers. Hence, consider the ith interconnected system [4]–[7], [10]

$$\dot{x}_i = A_i x_i + B_i u_i + G_i h_i(t,x), \quad y_i = C_i x_i, \qquad i = 1,2,3,\ldots,N \qquad (6)$$

where $x_i \in \Re^{n_i}$ are the states, $u_i \in \Re^{m_i}$ are the inputs, $y_i \in \Re^{p_i}$ are outputs, and $h_i(t,x)$ are the interconnections. It is assumed that (A_i, B_i) is stabilizable, (C_i, A_i) is detectable, and the uncertain term $h_i(t,x)$ is bounded by a quadratic inequality [4]–[7], [10]

$$h_i^T(t,x)h_i(t,x) \le \alpha_i^2 x^T H_i^T H_i x \qquad (7)$$

where $x = [x_1^T \; x_2^T \; \ldots \; x_N^T]^T$, $\alpha_i > 0$ are interconnection parameters, and H_i are known constant matrices. For the ith subsystem, the closed loop has the form

$$\dot{z}_{l_i} = \begin{bmatrix} A_i + B_i K_i & D_{l_i} C_i & C_{l_i} \\ 0 & A_i - D_{l_i} C_i & -C_{l_i} \\ 0 & B_{l_i} C_i & A_{l_i} \end{bmatrix} z_{l_i} + \begin{bmatrix} 0 \\ G_i \\ 0 \end{bmatrix} w_i = \hat{A}_{cl_i} z_{l_i} + G_{l_i} w_i \quad (8)$$

where $z_{l_i} = [\hat{x}_i^T \; e_i^T \; x_{l_i}^T]^T$ and $x_{l_i} \in \Re^{n_{l_i}}$ are the states of observer for the ith subsystem. Defining $z_N = [\hat{x}_1^T \; e_1^T \; x_{l_1}^T \; \ldots \; \hat{x}_N^T \; e_N^T \; x_{l_N}^T]^T$, the overall closed-loop system can be written as

$$\dot{z}_N = \hat{A}_D z_N + G_D w(t, z_N) \quad (9)$$

where $\hat{A}_D = \operatorname{diag}(\hat{A}_{cl_1}, \hat{A}_{cl_2}, \hat{A}_{cl_3}, \ldots, \hat{A}_{cl_N})$, $G_D = \operatorname{diag}(G_{l_1}, \ldots, G_{l_N})$, and $w = [w_1^T \; \ldots \; w_N^T]^T$, satisfying

$$w^T(t, z_N) w(t, z_N) \le z_N^T \left(\sum_{i=1}^{N} \frac{1}{\gamma_i^2} H_{i_l}^T H_{i_l} \right) z_N. \quad (10)$$

The elements of H_{i_l} corresponding to $x_{l_1}, \ldots, x_{l_N}$ are zero. In the following theorem, we provide sufficient conditions for the existence of stabilizing decentralized controllers.

Theorem 1: If the following optimization problem is feasible:

$$\min \sum_{i=1}^{N} \gamma_i,$$

subject to $\quad \Pi_{2_D}^T Y_D \Pi_{2_D} > 0$

$$\begin{bmatrix} \Pi_{2_D}^T \left(\hat{A}_D Y_D + Y_D \hat{A}_D^T \right) \times \Pi_{2_D} & \Pi_{2_D}^T G_D & \ldots & \Pi_{2_D}^T Y_D H_{N_l}^T \\ & -I & \ldots & 0 \\ & & \ddots & 0 \\ & & & \vdots \\ & & & -\gamma_N I \end{bmatrix} < 0$$

where Y_D is a block diagonal Lyapunov function and $\Pi_{2_D} = \operatorname{diag}(\Pi_{2_1}, \ldots, \Pi_{2_N})$ is a transformation matrix, then the system in (9) is asymptotically stable for all nonlinearities satisfying the quadratic constraint in (10).

Proof: Please see the Appendix.

Corollary 1: Under the condition of *Theorem 1*, if the quadratic constraint in (7) or (10) holds globally, then the system in (9) is globally stabilized.

It is interesting to note that [7] proposed an algorithm of decentralized output feedback control design for the class of nonlinear systems in (6). The method utilized the multifaceted tools of LMI, and the solution was obtained as a sequential two-part optimization problem. The control design was formulated as the following optimization problem [7]:

$$\min \sum_{i=1}^{N} \gamma_i, \quad \text{subject to} \quad Y > 0,$$

$$P_0 > 0 \quad \text{and} \quad \begin{bmatrix} F_c & S \\ & F_0 \end{bmatrix} < 0 \quad (11)$$

where F_c contains terms which are affine in decentralized controller parameters K_D and F_o contains that of decentralized observer parameters L_D. Here

$$S = \begin{bmatrix} -B_D K_D & 0 & \ldots & 0 \\ I & 0 & \ldots & 0 \end{bmatrix}$$

where $B_D = \operatorname{diag}(B_1, \ldots, B_N)$. Due to the presence of term $-B_D K_D$ in (11), this optimization problem is nonconvex. Therefore, to apply the LMI tools, [7] proposed the following two-step method.

Step 1) Compute K_D by solving $\min \sum_{i=1}^{N} \gamma_i$ subject to $Y > 0$ and $F_c < 0$.

Step 2) Using the K_D obtained from step 1), compute L_D by solving the following optimization problem:

$$\min \sum_{i=1}^{N} \beta_i \text{ subject to } P_0 > 0, \; \Lambda > 0, \; \begin{bmatrix} \Lambda F_c & S \\ & F_0 \end{bmatrix} < 0 \quad (12)$$

where the auxiliary parameter $\Lambda = \operatorname{diag}(\beta_1 I_1, \ldots, \beta_N I_N, \beta_1 I_1, \ldots, \beta_N I_N)$ has been added to make the optimization problem in the second step feasible.

This method has some merit as it provides a way to handle a nonlinear optimization problem which we often encounter in many control problems. Moreover, maximization of the interconnection bounds and the introduction of parameter Λ are interesting. However, our approach differs in the following ways.

- We formulated the decentralized controller and observer design problem as a convex optimization problem using different congruence transformations, simplifications, new variable definitions (modified form of [21]), and the use of Schur's complement method. An important property of the linear objective minimization problem in *Theorem 1* is that the set of all solutions are convex, thus it always gives the global minimum, if it exists.
- It should be noted that (11) and (12) are not theoretically equivalent. The constraint in (11) is equivalent to

$$\begin{bmatrix} \Lambda^{1/2} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} F_c & S \\ S^T & F_0 \end{bmatrix} \begin{bmatrix} \Lambda^{1/2} & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} \Lambda F_c & \Lambda^{1/2} S \\ S^T \Lambda^{1/2} & F_0 \end{bmatrix} < 0.$$

Λ in (12) has been introduced in a special way (only at the (1,1) position) to make the optimization problem feasible. This is because, after substituting the designed controller K_D from step 1) into step 2) (to compute the observer parameters with $\Lambda = I$), there is no guarantee of the feasibility of optimization problem. However

$$\begin{bmatrix} \Lambda F_c & S \\ S^T & F_0 \end{bmatrix} < 0 \Leftrightarrow$$

$$\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} \Lambda F_c & S \\ S^T & F_0 \end{bmatrix} \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} < 0 \Leftrightarrow \begin{bmatrix} F_0 & S^T \\ S & \Lambda F_c \end{bmatrix} < 0. \quad (13)$$

Using Schur's complement method [15], (13) is equivalent to $F_0 < 0$, $\Lambda F_c < 0$, and $F_0 - S^T (\Lambda F_c)^{-1} S < 0$. Therefore, by selecting β's sufficiently large, the feasibility of (13) can be guaranteed. Although, this method works well in many cases, control engineers still demands an explicit solution to this design problem. In *Theorem 1*, the control design problem has been converted into a convex optimization problem which can be directly solved using the LMI toolbox in one step. Hence, it should be useful to both practitioners and theorists.

- It is worthwhile to note that, due to the dynamic observer in our case, the number of LMI decision variables are $\sum_{i=1}^{N}[n_{l_i}\times n_{l_i}+n_{l_i}\times p_i+n_i\times\{n_{l_i}+m_i+p_i+(3(n_i+1)/2)\}]$. In [7], the number of decision variables are only $\sum_{i=1}^{N}[n_i\times(n_i+1)+n_i\times p_i+n_i\times m_i]$. An increase in the number of decision variables compared with static observer-based control design implies an increase in the off-line computational effort required to solve the LMIs. Moreover, the presence of controller dynamic equations require more online computations.

Remark 1: Theorem 1 does not place any constraint on the closed loop eigenvalues. There is a demand of constraining the position of closed-loop eigenvalues to fasten the observer dynamics, to avoid low-damped and/ or high-frequency modes, and to overcome very high feedback gain K_i, which the LMI formulation may produce. Therefore, after applying some congruence transformations and simplifications on the pole placement objectives of [21], the following constraints are added to bound the minimum decay rate, the maximum overshoot and the frequency of oscillatory modes for the control signal and the observation error dynamics, respectively:

$$\Pi_{2_i}^T\left(\hat{A}_{cl_i}Y_i+Y_i\hat{A}_{cl_i}^T\right)\Pi_{2_i}+2\alpha_{\min_i}\Pi_{2_i}^TY_i\Pi_{2_i}<0$$
$$\begin{bmatrix}-\omega_{n\max_i}\Pi_{2_i}^TY_i\Pi_{2_i} & \Pi_{2_i}^T\hat{A}_{cl_i}Y_i\Pi_{2_i}\\ & -\omega_{n\max_i}\Pi_{2_i}^TY_i\Pi_{2_i}\end{bmatrix}<0$$
$$\begin{bmatrix}(\mathcal{D}_{11})_i & (\mathcal{D}_{12})_i\\ & (\mathcal{D}_{11})_i\end{bmatrix}<0.$$

Here

$$\begin{aligned}(\mathcal{D}_{11})_i &=(\sin\theta)_i\left(\Pi_{2_i}^T\hat{A}_{cl_i}Y_i\Pi_{2_i}+\Pi_{2_i}^TY_i\hat{A}_{cl_i}^T\Pi_{2_i}\right)\\ (\mathcal{D}_{12})_i &=(\cos\theta)_i\left(\Pi_{2_i}^T\hat{A}_{cl_i}Y_i\Pi_{2_i}-\Pi_{2_i}^TY_i\hat{A}_{cl_i}^T\Pi_{2_i}\right)\\ \alpha_{\min_i} &=\mathrm{diag}\,[2(\alpha_{\min})_c,2(\alpha_{\min})_o,2(\alpha_{\min})_o]_i\\ \omega_{n\max_i} &=\mathrm{diag}\,[(\omega_{n\max})_c,\ (\omega_{n\max})_o,\ (\omega_{n\max})_o]_i\\ (\sin\theta)_i &=\mathrm{diag}\,[(\sin\theta)_c,\ (\sin\theta)_o,\ (\sin\theta)_o]_i\\ (\cos\theta)_i &=\mathrm{diag}\,[(\cos\theta)_c,\ (\cos\theta)_o,\ (\cos\theta)_o]_i\,.\end{aligned}$$

Subscript c refers to controller and o to observer.

Remark 2: It is interesting to note that the design algorithm in this section *recognizes* the matching conditions by returning an observer-based controller for any bound on the interconnection terms. Consider the system in (1) with $G=B$ and where $h_r(t,x)$ is replaced by $g(t,x)$, with $g(t,x)$ satisfying $g^T(t,x)g(t,x)\le\alpha^2x^TH_r^TH_rx$. It is well known that, in the presence of matching conditions, there always exists a stabilizing feedback control in spite of the size of perturbations [20]. It can be easily shown that our LMI formulation recognizes it, and a stabilizing observer-based controller can *always* be computed. This is also true for the interconnected system in (6). The proof is shown in the Appendix.

Remark 3: Decentralized dynamic observer-based controllers offer additional degrees of freedom in the design (compared with static ones). Also, the attractive feature of connective stability makes it appealing from an application point of view. To illustrate this point, consider the double pendulum example in [2], [4], and [20]. It is composed of two inverted penduli coupled by a sliding spring, where the position of the spring determines the extent of coupling among the two penduli. The state space model of this interconnected system is represented by [20]

$$\begin{aligned}\dot{x}_1 &=\begin{bmatrix}0&1\\1&0\end{bmatrix}x_1+\begin{bmatrix}0\\1\end{bmatrix}u_1+\begin{bmatrix}0&0&0&0\\-1&0&1&0\end{bmatrix}e(t,x)x\\ y_1 &=[1\quad 0]x_1\\ \dot{x}_2 &=\begin{bmatrix}0&1\\1&0\end{bmatrix}x_2+\begin{bmatrix}0\\1\end{bmatrix}u_2+\begin{bmatrix}0&0&0&0\\1&0&-1&0\end{bmatrix}e(t,x)x\\ y_2 &=[1\quad 0]x_2.\end{aligned}\tag{14}$$

Here, $x_1=[x_{11}^T\ x_{12}^T]^T$ represents the angular position and the angular velocity of the first pendulum and x_2 represents that of the second. The normalized interconnection parameter $e:\Re^5\to[0,1]$ represents the degree of coupling among the two penduli and is useful to study connective stability [4] of the penduli under structural perturbations caused by the jumps of the coupled spring. Our control objective is to design decentralized control laws to keep the penduli in an upright position, robustly stabilize the system in (14) for $e(t,x)\in[0,1]$, while at the same time maximize the uncertainty bounds α_1 and α_2 on the interconnections $h_1(t,x)$ and $h_2(t,x)$. Now, it is clear that

$$\begin{aligned}h_1^T(t,x)h_1(t,x) &\le x^T\begin{bmatrix}1&0&-1&0\\0&0&0&0\\-1&0&1&0\\0&0&0&0\end{bmatrix}x\\ h_2^T(t,x)h_2(t,x) &\le x^T\begin{bmatrix}1&0&-1&0\\0&0&0&0\\-1&0&1&0\\0&0&0&0\end{bmatrix}x.\end{aligned}\tag{15}$$

Since (A_i,B_i) is stabilizable and (C_i,A_i) is detectable, the LMI formulation of *Theorem 1* can be used to design decentralized observer-based controllers. By appropriately selecting an admissibility region as

$$\begin{aligned}(\alpha_{\min})_c &=0.5\\ (\omega_{n\max})_c &=6.3\\ (\theta)_c &=\frac{\pi}{4}\\ (\alpha_{\min})_0 &=2.5\\ (\omega_{n\max})_o &=10\\ (\theta)_o &=\frac{\pi}{4}\end{aligned}\tag{16}$$

for both subsystems and solving the linear objective minimization problem in *Theorem 1*, we get

$$\begin{aligned}&\alpha_1=\alpha_2=0.581\\ &K=\begin{bmatrix}-30.60&-9.23&0&0\\0&0&-30.60&-9.23\end{bmatrix}\\ &R_{1_1}=R_{1_2}=\begin{bmatrix}4.61&-23.2\\-23.2&175.7\end{bmatrix}\\ &X_{r_1}=X_{r_2}=\begin{bmatrix}1.47&-4.12\\-4.12&20.58\end{bmatrix}\\ &Y_{r_1}=Y_{r_2}=\begin{bmatrix}11.35&-1.37\\-1.37&0.397\end{bmatrix}.\end{aligned}\tag{17}$$

TABLE I
EIGENVALUES OF THE CLOSED-LOOP SYSTEM

$-4.61 + j\,2.88$	$-5.7 + j\,3.24$	-2.86
$-4.61 - j\,2.88$	$-5.7 - j\,3.24$	-9.59

The parameters $R_{1_1}(R_{1_2})$, $X_{r_1}(X_{r_2})$, and $Y_{r_1}(Y_{r_2})$ determine the positive definite Lyapunov function $\Pi_{2_D}^T Y_D \Pi_{2_D}$, which guarantees robust stability of the system in (14) for the bound on the interconnection $|e(t,x)| \le 0.581$. As expected, the individual decoupled closed-loop subsystems, which are obtained from (14) by using K of (17) and setting $h_i(t,x) \equiv 0$, are all stable for $i = 1{,}2$, thereby revealing *connective* stability. The admissibility region in (16) has been carefully selected to obtain a good transient characteristic. The eigenvalues of $\hat{A}_{cl_1}$ and $\hat{A}_{cl_2}$ are tabulated in Table I. The first two eigenvalues are responsible for the controller dynamics and the remaining four are responsible for the observer dynamics. Clearly, the eigenvalues lie in the region specified by (16). For different initial conditions, Fig. 1 shows the dynamics of the states and the estimation errors, respectively. Clearly, the controller in (17) stabilizes the overall interconnected system. However, the value of α's in (17) reveals that robust stability can be only guaranteed for $e(t,x) \in [0, 0.581]$. In [4], the model of this inverted pendulum was also used and static observer-based controllers were designed with $B_i = I_i$, where I_i is the $n_i \times n_i$ identity matrix. Therefore, the model was not realistic, and the assumption that the number of control inputs are equal to the dimension of state was very restrictive.

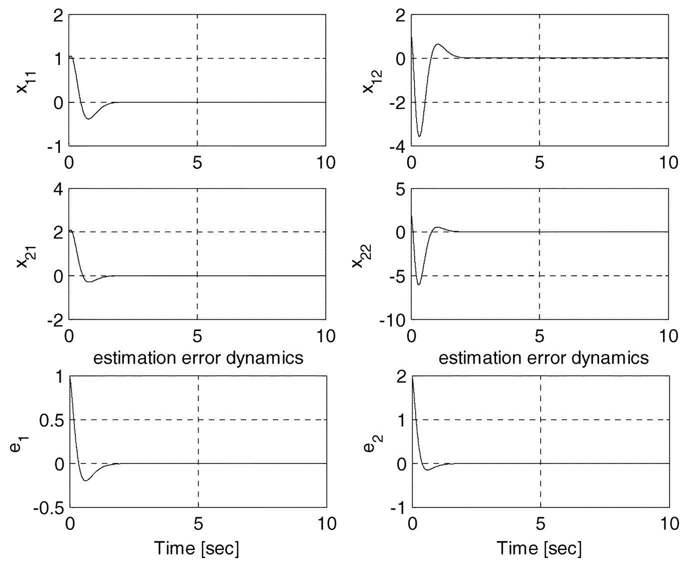

Fig. 1. Stabilizing effect of the decentralized controller.

III. APPLICATION TO INDUSTRIAL BOILER SYSTEMS

Here, we extend the nonlinear model of [19] to include the dynamics of the governor, turbine, and the generator unit. Starting from the nonlinear model in [19], some simplifications give [10]

$$
\begin{aligned}
\dot{x}_1 &= \frac{(e_{22} - e_{12}h_f)u_1 + (e_{12}h_s - e_{22})u_2 - e_{12}u_3/h_m}{e_{11}e_{22} - e_{12}e_{21}} \\
\dot{x}_2 &= \frac{(e_{11}h_f - e_{21})u_1 + (e_{21} - e_{11}h_s)u_2 + e_{11}u_3/h_m}{e_{11}e_{22} - e_{12}e_{21}} \\
\dot{x}_3 &= -\frac{h_c q_{dc} x_3}{e_{33}} + \frac{e_{21}e_{32} - e_{11}e_{32}h_f}{e_{11}e_{22}e_{33} - e_{12}e_{21}e_{33}} u_1 \\
&\quad + \frac{(e_{11}e_{32}h_s - e_{21}e_{32})u_2 + (e_{11}e_{22} - e_{12}e_{21})u_3/h_m}{e_{11}e_{22}e_{33} - e_{12}e_{21}e_{33}} \\
\dot{x}_4 &= \frac{e_{43}h_c q_{dc} x_3}{e_{33}e_{44}} - \frac{\rho_s x_4}{T_d e_{44}} + \frac{\rho_s x_4^0}{e_{44}T_d} \\
&\quad + \frac{e_{21}e_{32}e_{43}u_2}{(e_{11}e_{22} - e_{12}e_{21})e_{33}e_{44}} - \frac{e_{43}u_3/h_m}{e_{33}e_{44}}.
\end{aligned} \tag{18}
$$

The state variables of this system are: total water volume (x_1), drum pressure (x_2), steam-mass fraction in risers (x_3), and steam volume in the drum (x_4). The control variables are feedwater flow rate (u_1), steam flow rate (u_2), and fuel flow rate (u_3). Here, $e_{11}, \ldots, e_{44}$ are functions of the states $(x_1, \ldots, x_4)$, the construction parameters, and the properties of steam [19]. The parameters h, ρ, and q_{dc} represent specific enthalpy, specific density, and downcomer flow rate, respectively. Subscripts s, w, and m stand for steam, water, and methane, respectively. Linearization of (18) shows that the system has two poles at the origin, one associated with water dynamics and the other with pressure dynamics [10], [19]. In the following, we move the pole associated with the pressure dynamics to the left half plane. This is done by connecting the boiler drum to a governor, turbine, and generator unit (see Fig. 2).

We utilized ideas from [19] and [22] and started with some experiments that can be used as a basis of modeling. In the experiment, the variables such as the feedwater flow, fuel flow, and the control valve position (X) are considered as inputs. The recorded outputs are drum level, drum pressure, steam flow, and the active power output (P_m). It has been found that the drum pressure responds in more or less the same way to changes in fuel flow and control valve opening. However, the response of active power output to step change in the control valve setting shows very fast change. Also, the drum pressure decreases with the increase in valve opening. However, these variations are not uniform with the corresponding change in input. Many other experiments also reveals that the system is highly *interacting* and *nonlinear*. As the drum pressure is a significant measure of the state of boiler, if the mass content of the drum is constant, and the distribution of energy stored in iron, water, and steam masses do not change during transients, then the stored energy can be expressed as [22] $E = ax_2 + b$. With this assumption, we have $dE/dt = a(dx_2/dt) = P_i - P_m$, where P_i is the input power and P_m is the output power of the boiler–turbine unit. As the output power P_m is a function of control valve position and the pressure at turbine, we can write [22]

$$P_m = r_1 u_2 \Delta h + r_2 \tag{19}$$

where Δh is the enthalpy drop across the turbine, and r_2 is added to account for losses in flow. Moreover, as the input power is a function of feedwater flow and fuel flow rate, P_i can be expressed as $P_i = a_1 u_3 - a_2 u_1$. From the figure of steam flow u_2 versus drum pressure x_2, we obtain $u_2 = 0.0904 \times 10^{-3} X x_2 - 538.9242$, where x_2 is in Pascal. In power-generating systems, the main part of power (70%) is generated by the intermediate and low-pressure parts of the turbine. Therefore, we estimated

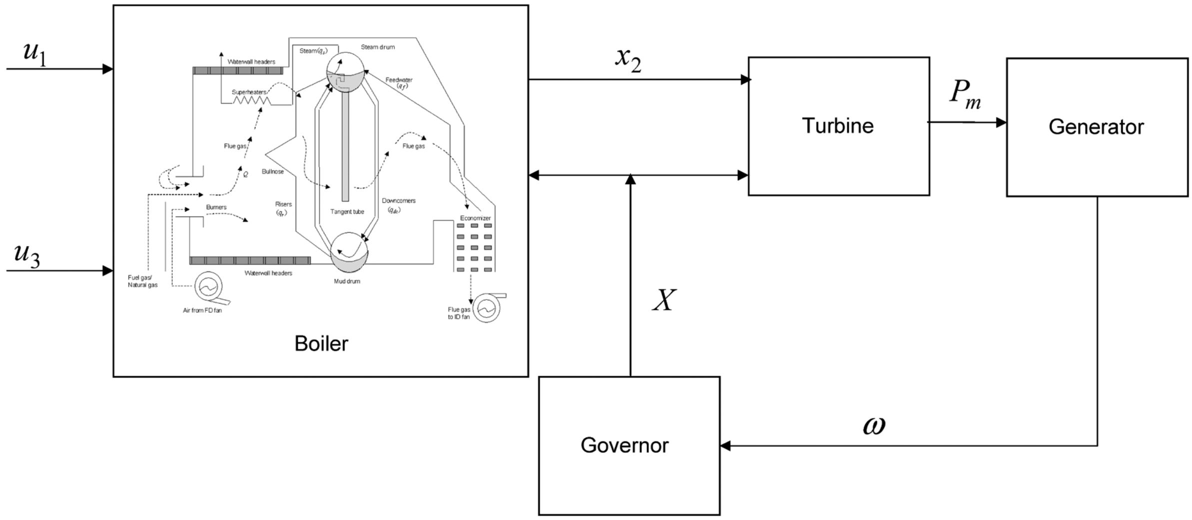

Fig. 2. Schematic of the boiler, governor, and turbine unit.

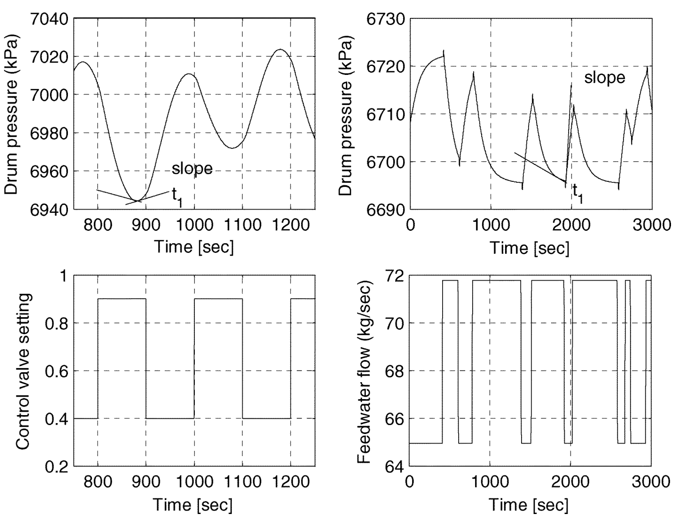

Fig. 3. Computation of parameters α_1 and α_3.

the output power from the enthalpy drop in pressure interval 1 MPa to 4 MPa, which gives $\Delta h \approx 103.99x_2^{1/8}$. Combining all, (19) reduces to $P_m = \alpha_4(Xx_2^{9/8} - \alpha_5)$, and

$$\frac{1}{a}\frac{dE}{dt} = \frac{dx_2}{dt} = \alpha_2 u_3 - \alpha_3 u_1 - \alpha_1\left(Xx_2^{9/8} - \alpha_5\right) \quad (20)$$

where $\alpha_1 = \alpha_4/a$, $\alpha_2 = a_1/a$, and $\alpha_3 = a_2/a$. Fig. 3 shows the dynamics of drum pressure, when small perturbations are done on the steam valve opening and feedwater flow. Keeping u_1 and u_3 constant, a change $\Delta\dot{x}_2$ of dx_2/dt at time t_1 due to change ΔX of X is $\Delta\dot{x}_2(t_1) = -\alpha_1 x_2^{9/8}(t_1)\Delta X(t_1)$ From the figure, we get $\alpha_1 = 2.876 \times 10^{-6}$. To compute α_3, small perturbations are done on feedwater flow (see Fig. 3) and $\alpha_3 = 26.75$ Pa/kg is achieved. Similar arguments give $\alpha_2 = 1019$ and $\alpha_5 = -9.16$. After getting the parameters and substituting u_2 (in terms of x_2 and X), (18) and (20) clearly reveals that the pole associated with the pressure dynamics has moved to the left half of complex plane. A Taylor-series expansion of (18) is then done to bring it in the standard form. As the steam mass fraction is always less than 100%, defining a region: $\Omega = \{x : x_1, x_4 \in \Re, |x_3| \leq 1, x_2 \leq 7.1 \text{ MPa}, |X| \leq 0.5\}$ and denoting x_0 as the operating point, we have

$$\begin{aligned}
h_1^T(x)h_1(x) \leq &\Bigg[\frac{1.3(\rho_s - \rho_w)q_{dc}}{\rho_s[(1-x_3)\rho_s + x_3\rho_w]} \\
&\quad\times\Bigg\{1 + \frac{x_3(\rho_s-\rho_w)}{[(1-x_3)\rho_s + x_3\rho_w]} \\
&\qquad + \frac{x_3^2(\rho_s-\rho_w)^2}{[(1-x_3)\rho_s + x_3\rho_w]^2}\Bigg\}\Bigg]_{x_0}^2 \Delta x_3^2 \\
&+ \Bigg[\frac{2\rho_s q_{dc}(\rho_s-\rho_w)}{V_r\rho_w[\rho_s - x_3\rho_w]^2} \\
&\quad\times\Bigg\{1 + \frac{x_3\rho_w}{[\rho_s - x_3\rho_w]}\Bigg\}\Bigg]_{x_0}^2 \Delta x_3^2 \\
&+ 1.4641\Delta X^2, \quad \forall x \in \Omega.
\end{aligned}$$

Normally, the boiler effect (B_{eff}) is represented by the drum pressure, and the governor effect (G_{eff}) is represented by the control valve position. Hence, in order to match the boiler and turbine models, the input to turbine is calculated from steady-state power ($B_{\text{eff}} \times G_{\text{eff}} = 3.12Xx_2^{9/8}$). The physical model of the governor, turbine, and generator is given by

$$\begin{aligned}
\dot{\delta} &= \omega \\
\dot{\omega} &= -\frac{\omega}{T_p} + \frac{K_p}{T_p}\left[P_m = 0.3\left(P_{m_h} + P_{m_L}\right) + 0.4P_{m_I}\right] \\
0.35\dot{P}_{m_h} &= -P_{m_h} + 3.12Xx_2^{9/8} \\
\dot{P}_{m_I} &= -\frac{P_{m_I}}{7} + \frac{P_{m_h}}{7} \\
\dot{P}_{m_L} &= -\frac{P_{m_L}}{0.5} + \frac{P_{m_I}}{0.5} \\
T_E\dot{X} &= -X - \frac{\omega}{R} + P_m^0 + u
\end{aligned}$$

where P_{m_h}, P_{m_I}, and P_{m_L} are mechanical power outputs of the high-pressure, intermediate-pressure, and low-pressure turbines, respectively. R is the regulation constant, T_E and T_p are

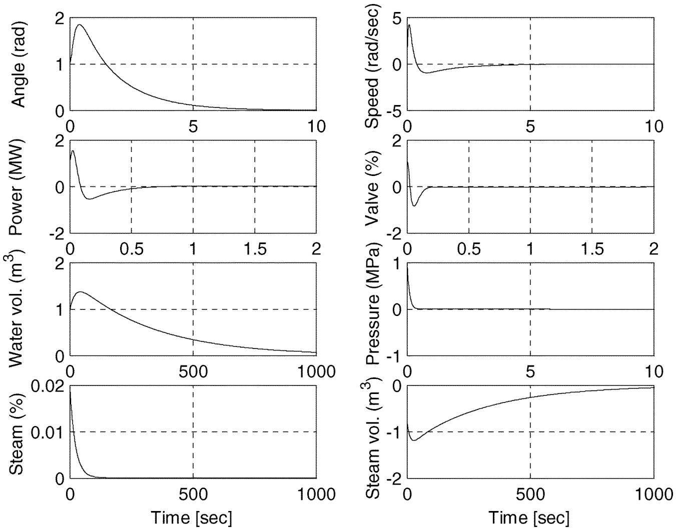

Fig. 4. Stabilizing effect of the controller.

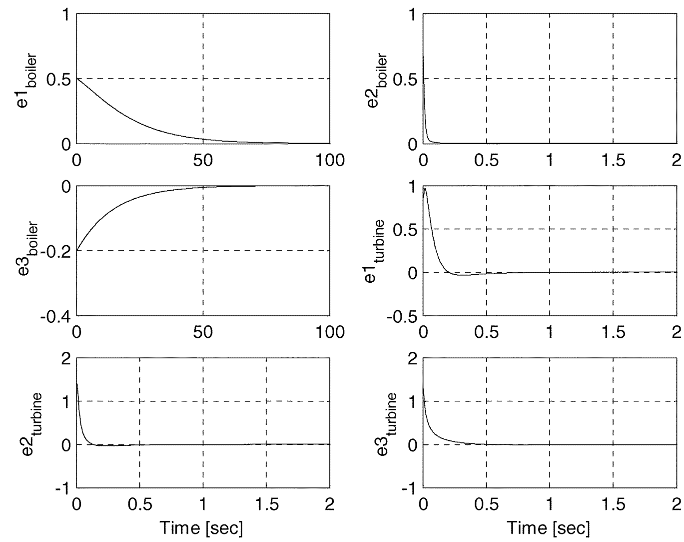

Fig. 5. Estimation error dynamics.

time constants, K_P is the power system gain, u is the control input, δ is the power angle, ω is the angular speed, and P_m is the total mechanical power output.

It is important to note that $X = 0$ corresponds to a 50% valve opening. Hence, -0.5 corresponds to a fully closed valve and $+0.5$ corresponds to a fully opened valve. Also, as the steam mass fraction is always less than 100%, the bound $|x_3| \leq 1$ is used in Ω. According to field experience, the utility boiler works under three different load conditions (low load, normal load, and high load), and the drum pressure always remain less than 7.1 MPa. Therefore, although the nonlinear model of the steam-generating and electricity-generating unit contains quadratic nonlinearities (e.g., $x_2^{9/8}$ and x_3^4), it is possible to bound the nonlinear functions as in (7) after some linear algebra. Moreover, even if the quadratic bounds are computed locally (limits of X, x_3, and x_2), in reality, the physical limits of these state variables lie in this range and the presented method can work well in different load conditions.

Control Strategy: After completing the modeling part, our control strategy is to decompose the plant into two subsystems: 1) boiler and 2) governor, turbine, and generator unit. They are interconnected as in Fig. 2. The overall linearized system has two poles at the origin: one associated with water dynamics in boiler and the other with generator dynamics. Moreover, the electrical unit has also eigenvalues at $0.9359 \pm 8.0259i$. The stabilizing decentralized controller is designed by solving the linear objective minimization problem of *Theorem 1*. It is given by: $K_{\text{turbine}} = [-0.2671 \ -0.4325 \ -1.0595 \ -1.7186 \ -1.0506 \ -4.4705]$ and

$$K_{\text{boiler}} = \begin{bmatrix} -119.48 & 0.000253 & -7391.596 & -152.91 \\ -3.136 & -0.0121 & -194.039 & -4.0141 \end{bmatrix}.$$

The stabilizing effect of controllers are shown in Fig. 4. It can be seen that the dynamics of electricity generating unit is much faster than the drum-boiler dynamics. The term “0” on the y-axis represents steady-state values (e.g., $X^0 = 0.5$ and $P_m^0 = 48$ MW). Fig. 5 shows the dynamics of estimation error, where $(e_1)_{\text{boiler}}$, $(e_2)_{\text{boiler}}$, and $(e_3)_{\text{boiler}}$ represent $x_1 - \hat{x}_1$, $x_2 - \hat{x}_2$, and $x_4 - \hat{x}_4$, respectively. The others are for the turbine-generator unit (δ, ω, and P_m).

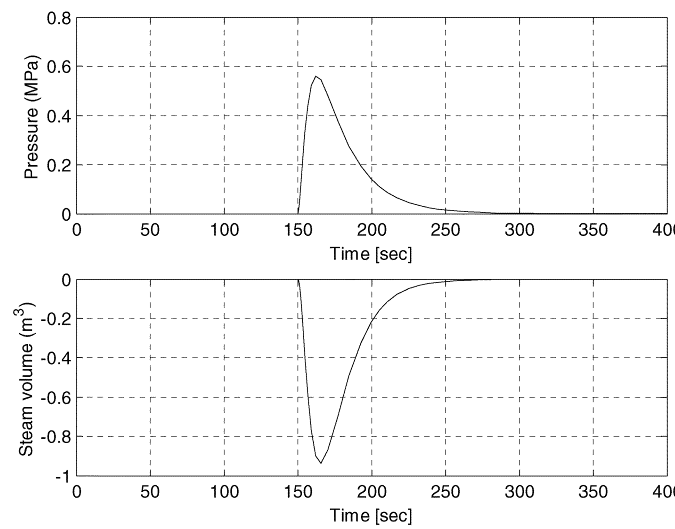

Fig. 6. Disturbance rejection.

Fig. 6 shows the disturbance rejection capability of the boiler controller caused by sudden load variations in the system. Due to a sudden decrease in the load, the steam valve opening reduces and the pressure in the drum suddenly rises. This increases the volume of water due to increased condensation. The volume of the steam in the drum decreases due to the increased pressure, which causes condensation of the steam. The controller is also capable of attenuating the effect of a sudden increase in load.

IV. Conclusion

In this paper, we proposed a new algorithm for decentralized observer-based control design, which guarantees robust stabilization of the overall interconnected system. The approach alleviates the necessity of having invertible input matrix B_i [4], choice of parameters by trial and error [6], or any two-step approach [7]. The design utilizes the versatile tools of LMI, which has the flexibility of accommodating various design constraints involving matching conditions, separate regions of controller and observer eigenvalues, and the bounds on nonlinear terms. It is useful to know that the LMI formulation maximizes the interconnection bounds, thereby increasing the robustness of the closed-loop system against uncertain nonlinear interconnections. In addition to this, the control law results in a connectively stable system, thus overcoming the barrier of instability caused by sudden structural perturbations.

The developed theoretical framework is then applied to a power unit. We extend the model of [19] to incorporate the dynamics of governor, turbine, and generator. Simulation results show that the stabilizing effect of controllers are good both under normal and perturbed conditions, which make it practically implementable.

APPENDIX

Proof of Theorem 1: Here, we first develop a control design algorithm for the autonomous system in (1), which is then generalized to multiple subsystems in (6). Let us consider a Lyapunov function $v = z^T P z$ [14], where P is a symmetric positive definite matrix ($P > 0$). The sufficient conditions for asymptotic stability of the closed-loop system in (4) are

$$\begin{aligned} P &> 0 \\ z^T \hat{A}^T P z + w^T G_r^T P z + z^T P \hat{A} z + z^T P G_r w &< 0. \end{aligned}$$

According to the S-procedure [15], when (5) is satisfied, the above condition is equivalent to the existence of matrix P and a number $\tau > 0$ such that $P > 0$ and

$$\begin{bmatrix} \hat{A}^T P + P\hat{A} + \tau\alpha^2 H^T H & PG_r \\ G_r^T P & -\tau I \end{bmatrix} < 0. \tag{21}$$

This is equivalent to the existence of matrix $Y > 0$, and

$$\begin{bmatrix} \hat{A}Y + Y\hat{A}^T & G_r & YH^T \\ G_r^T & -I & 0 \\ HY & 0 & -\gamma I \end{bmatrix} < 0 \tag{22}$$

where $Y = \tau P^{-1}$ and $\gamma = 1/\alpha^2$. This LMI cannot be used to find the observer-based controller because it is not affine on observer parameters A_l, B_l, C_l, and D_l, which are constant $n_l \times n_l$, $n_l \times p$, $n \times n_l$, and $n \times p$ matrices, respectively. Hence, a variable transformation is necessary. Partitioning Y and Y^{-1} as

$$Y = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & X_r & M_r \\ 0 & M_r^T & V_r \end{bmatrix}$$
$$Y^{-1} = \begin{bmatrix} R_2 & 0 & 0 \\ 0 & Y_r & N_r \\ 0 & N_r^T & U_r \end{bmatrix}$$

where R_1, R_2, X_r, and Y_r are $n \times n$ and symmetric, M_r and N_r are $n \times n_r$, and $Y > 0$ implies $R_1 > 0$, $X_r > 0$, and $Y_r > 0$. From $Y^{-1}Y = I$, it can be inferred that $Y^{-1}[R_1^T\ X_r^T\ M_r]^T = [I\ I\ 0]^T$, which leads to $Y^{-1}\Pi_1 = \Pi_2$, where

$$\Pi_1 \triangleq \begin{bmatrix} R_1 & 0 & 0 \\ 0 & X_r & I \\ 0 & M_r^T & 0 \end{bmatrix}$$
$$\Pi_2 \triangleq \begin{bmatrix} I & 0 & 0 \\ 0 & I & Y_r \\ 0 & 0 & N_r^T \end{bmatrix}. \tag{23}$$

Pre-and post-multiplying $Y > 0$ by Π_2^T and Π_2, respectively, and (22) by $\mathrm{diag}(\Pi_2^T, I, I)$ and $\mathrm{diag}(\Pi_2, I, I)$, respectively, we have $\Pi_2^T Y \Pi_2 > 0$ and

$$\begin{bmatrix} \Pi_2^T \hat{A} Y \Pi_2 + \Pi_2^T Y \hat{A}^T \Pi_2 & \Pi_2^T G_r & \Pi_2^T Y H^T \\ G_r^T \Pi_2 & -I & 0 \\ HY\Pi_2 & 0 & -\gamma I \end{bmatrix} < 0. \tag{24}$$

Some linear algebra shows that

$$\Pi_2^T Y \Pi_2 = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & X_r & I \\ 0 & I & Y_r \end{bmatrix}$$
$$\Pi_2^T \hat{A} Y \Pi_2 = \begin{bmatrix} AR_1 + BL_1 & \hat{C}_r & \hat{D}_r C \\ 0 & AX_r - \hat{C}_r & A - \hat{D}_r C \\ 0 & \hat{A}_r & Y_r A + \hat{B}_r C \end{bmatrix}$$

where

$$\begin{aligned} \hat{A}_r &\triangleq Y_r^T (A - D_l C) X_r + N_r B_l C X_r - Y_r^T C_l M_r^T + N_r A_l M_r^T \\ \hat{B}_r &\triangleq -Y_r D_l + N_r B_l \\ \hat{C}_r &\triangleq D_l C X_r + C_l M_r^T \\ \hat{D}_r &\triangleq D_l \\ L_1 &\triangleq K R_1. \end{aligned} \tag{25}$$

Also

$$\Pi_2^T \hat{A} Y \Pi_2 + \Pi_2^T Y \hat{A}^T \Pi_2 = \begin{bmatrix} \mathcal{F}_{11} & \hat{C}_r & \hat{D}_r C \\ \hat{C}_r^T & \mathcal{F}_{22} & \mathcal{F}_{23} \\ C^T \hat{D}_r^T & \mathcal{F}_{23}^T & \mathcal{F}_{33} \end{bmatrix}$$
$$\Pi_2^T G_r = \begin{bmatrix} 0 \\ G \\ Y_r G \end{bmatrix}$$
$$\Pi_2^T Y H^T = \begin{bmatrix} R_1 H_r^T \\ X_r H_r^T \\ H_r^T \end{bmatrix}$$

where $\mathcal{F}_{11} \triangleq AR_1 + BL_1 + R_1 A^T + L_1^T B^T$, $\mathcal{F}_{22} \triangleq AX_r + X_r A^T - \hat{C}_r - \hat{C}_r^T$, $\mathcal{F}_{23} \triangleq A + \hat{A}_r^T - \hat{D}_r C$, and $\mathcal{F}_{33} \triangleq Y_r A + \hat{B}_r C + A^T Y_r + C^T \hat{B}_r^T$. This makes the LMIs in (24) affine in controller and observer parameters. A linear objective minimization problem $(\min \gamma)$ subject to the convex constraints in (24) can be easily applied to compute them.

Since X_r and Y_r are symmetric matrices, N_r and M_r can be chosen to be square and nonsingular such that $N_r M_r^T = I - Y_r X_r$. Using singular value decomposition, we have $[\Sigma\Lambda\Omega^T] = \mathrm{svd}(I - Y_r X_r)$. This gives $N_r M_r^T = \Sigma\Lambda\Omega^T$, $N_r = \Sigma\Lambda^{1/2}$, and

$M_r = \Omega\Lambda^{1/2}$. Hence, from (25), the parameters K, A_l, B_l, C_l, and D_l can be easily calculated as

$$\begin{aligned} K &= L_1 R_1^{-1} \\ D_l &= \hat{D}_r \\ C_l &= (\hat{C}_r - D_l C X_r)\left(M_r^T\right)^{-1} \\ B_l &= N_r^{-1}(\hat{B}_r + Y_r D_l) \\ A_l &= N_r^{-1}\Big(\hat{A}_r - Y_r^T A X_r + Y_r^T D_l C X_r \\ &\qquad - N_r B_l C X_r + Y_r^T C_l M_r^T\Big)\left(M_r^T\right)^{-1}. \end{aligned} \tag{26}$$

This method does not require that the input matrix B be invertible [4] or necessitate the choice of any parameters by trial and error [6]. Hence, the critical restriction is removed. One can easily verify that, when $A_l = B_l = C_l = 0$ (static observer gain), then $\hat{A}_r = Y_r^T(A - D_l C)X_r$, $\hat{B}_r = -Y_r D_l$, $\hat{C}_r = D_l C X_r$, and $\hat{D}_r = D_l$. Hence, the parameters become nonaffine.

Now, the decentralized control design for the system in (6) is quite straightforward with block diagonal Lyapunov function $Y_D = \mathrm{diag}(Y_1, \ldots, Y_N)$ and transformation matrix $\Pi_{2_D} = \mathrm{diag}(\Pi_{2_1}, \ldots, \Pi_{2_N})$. Similar to (22), it is easy to verify that the sufficient conditions of the closed-loop system in (9) to be asymptotically stable under the constraint in (10) are $Y_D > 0$ and

$$\begin{bmatrix} \hat{A}_D Y_D + Y_D \hat{A}_D^T & G_D & Y_D H_{1_l}^T & \ldots & Y_D H_{N_l}^T \\ & -I & 0 & \ldots & 0 \\ & & -\gamma_1 I & \ldots & 0 \\ & & & \ddots & \vdots \\ & & & & -\gamma_N I \end{bmatrix} < 0.$$

Pre- and post-multiplying $Y_D > 0$ by $\Pi_{2_D}^T$ and Π_{2_D}, respectively, and the above constraint by $\mathrm{diag}(\Pi_{2_D}^T, I, I)$ and $\mathrm{diag}(\Pi_{2_D}, I, I)$, respectively, the LMIs become affine in controller and observer parameters.■

Proof of Matching Condition: Assume that H_r has full rank, i.e., $H_r^T H_r$ is positive definite and (A, H_r) is detectable [20]. With $G = B$, some constraints in the LMI formulation of (24) are $\gamma > 0$, $R_1 > 0$, $X_r > 0$, $Y_r > 0$, and

$$\begin{aligned} A X_r + X_r A^T - \hat{C}_r - \hat{C}_r^T + B B^T + \frac{X_r H_r^T H_r X_r}{\gamma} &< 0 \\ Y_r A + \hat{B}_r C + (Y_r A + \hat{B}_r C)^T + Y_r B B^T Y_r + \frac{H_r^T H_r}{\gamma} &< 0 \\ (A + BK)R_1 + R_1(A + BK)^T + \frac{R_1 H_r^T H_r R_1}{\gamma} &< 0. \end{aligned}$$

The feasibility of the first inequality (<0) can be easily proved using the approach in [20]. To prove the feasibility of the second inequality, consider the Riccati equation $A^T Y_m + Y_m A + Y_m B B^T Y_m + ((H_r^T H_r + \epsilon C^T C)/\bar{\gamma}) = 0$, where $\epsilon > 0$ and $0 < \bar{\gamma} < \gamma$. Since (A, H_r) and (A, C) are detectable, there always exists a unique positive definite solution Y_m to the Riccati equation such that $(A - B B^T Y_m)$ is a stable matrix. Therefore, by choosing $Y_r = Y_m$ and $\hat{B}_r = \epsilon C^T / 2\gamma$, we can get a solution to the LMI for any $\gamma > 0$. Here, ϵ determines the convergence rate of observer. Finally, to prove feasibility of the third inequality, consider the Lyapunov equation $A_c^T R + R A_c + (H_r^T H_r / \bar{\gamma}) = 0$, where A_c has all eigenvalues in the left half of the s-plane. Since $H_r^T H_r > 0$, A_c is stable, and $R^{-1} A_c^T + A_c R^{-1} + (R^{-1} H_1^T H_1 R^{-1} / \gamma) < 0$, the choice of $R_1 = R^{-1}$ and $A + BK = A_c$ leads to the solution of LMI for any $\gamma > 0$. ■

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