Elsevier, 2006
ISBN: 978-0-12-088400-1
USD$75.95.

Visualizing Quaternions

by ANDREW J. HANSON

Visualizing Quaternions presents the properties of quaternions and their applications. The pedagogy employed is a marriage of three-dimensional visualization with tractable mathematics. The text is arranged into three parts. Part I, which consists of 14 chapters, reviews the fundamental properties of quaternions; Part II, which also consists of 14 chapters, presents methodologies for utilizing quaternions in an efficient and effective manner; and Part III uses three chapters to challenge the reader to generalize the properties of quaternions to higher dimensional spaces.

For several years the computer graphics, kinematics, and geometry communities have lacked a comprehensive text that presents quaternions in an accessible and focused manner. *Visualizing Quaternions* satisfies that need. Quaternions have become an indispensible tool for representing the orientation of bodies, realizing rotational motion, and interpolating rotations. By presenting a sound exposition of quaternions in conjunction with the visualization of their inherent three (and even *four!*) dimensionality, *Visualizing Quaternions* is a success.

Visualizing Quaternions is unique in its presentation of quaternions with respect to the related texts [1]–[4]. Kuiper's text [1] elegantly presents quaternions and rotation operators from a mathematics perspective but lacks the visualization and software implementation that is included in *Visualizing Quaternions*. Although [2] and [3] present sound discussions of quaternions, these works are general kinematics texts that have broader scopes. Nevertheless, the interested reader of *Visualizing Quaternions* would be well served to review these alternative presentations. Whereas [4] presents a complete and formal presentation of the subject of spatial rotations, *Visualizing Quaternions*, by virtue of focusing on quaternions, is able to present the material in a modern and accessible fashion that is very appealing. Additional historical accounts of the development of quaternions are found in [5] and [6]. Other related works include [7], which presents an algebraic approach to the study of quaternions with applications to physics, and [8], which lucidly presents the geometry of quaternion and octonion algebras.

Digital Object Identifier 10.1109/MCS.2008.925869

CONTENTS

Part I begins with two of the strengths of this book, the motivational and historical chapters. To the uninitiated, quaternions are often a mysterious and abstract mathematical entity, having Chapter 2 dedicated to motivating the study of quaternions with sound physical and computer graphics examples brings the subject to life. Moreover, the author's recounting of the history of Sir W.R. Hamilton's obsession to generalize complex numbers to three-dimensional space that eventually led to his discovery of quaternions in Chapter 1 is entertaining and places the development of quaternions within the modern history of mathematics.

Part II, "Advanced Quaternion Concepts," makes up the bulk of the text. The level of mathematics utilized is a step above that in Part I, yet the author succeeds in maintaining the level of accessibility of the material. These 14 chapters present the properties of quaternions and algorithms for their use. While some are well known in the research community, many of these algorithms and properties will be new to even the accomplished practitioner. Included are classical Frenet-Serret frames, quaternion frames, quaternion volumes, quaternion interpolation, a review of rotation groups, spherical Riemannian geometry, and induced metrics on spheres.

In the 38 pages that make up Part III, titled "Beyond Quaternions," the book generalizes the properties of quaternions to higher dimensional spaces, for example, Clifford algebras and octonions. This part of the text is accessible to advanced graduate students and well-versed practitioners.

The numerous color illustrations provided in the book and the demonstration software codes and other materials

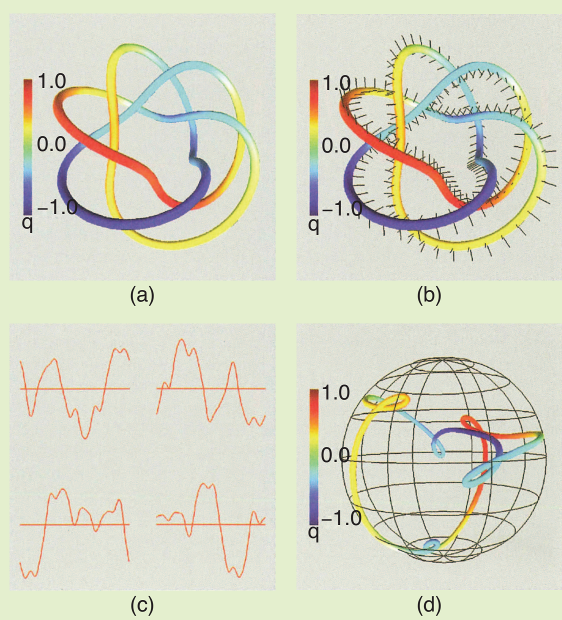


FIGURE 1. Figure 20.14 from *Visualizing Quaternions*. Reprinted with permission from the publisher.

1066-033X/08/$25.00©2008IEEE

available at the author's companion Web site (www.visualizingquaternions.com) are effective supplements to the printed text. For example, in Figure 1, *Visualizing Quaternions* presents the visualization of the Frenet frames assigned to a three-dimensional (3,5) torus knot. The frames are assigned by using the algorithm included in Section 20.7.1. Image (d) of the figure shows the projection of the quaternion frame components onto the 3-sphere.

The organization of the material into small concise chapters, where the average length of the 32 chapters is 12 pages, facilitates the use of *Visualizing Quaternions* as a reference. However, with some effort on the part of the instructor, the text could be used to support a course. Part I is suitable for a junior/senior level course, while parts I and II combined would be suitable for a first-year graduate-level course. Such courses would be appropriate electives for students studying computer sciences, physics, geometry, aerospace control, and robotics. Two challenges to the instructor in using the text in a course would be the lack of problems and the mixture of software languages included in the examples and companion website, for example, C and Mathematica.

A useful inclusion in *Visualizing Quaternions* is an extensive appendix, 51 pages long, that consists of the equations and algorithms presented in the text as well as some useful related material. The appendix presents these materials in a layout that facilitates the writing of software to implement the algorithms found in the text.

Visualizing Quaternions concludes with Chapter 32, which in two pages eloquently restates the simplicity, beauty, and utility of the quaternion. For those already familiar with quaternions, I suggest reading this chapter first. From it you will quickly garner an appreciation of the author's passion for the subject and his collegial writing style.

CONCLUSIONS

Though targeted at the computer graphics community, kinematicians, geometers, aerospace flight dynamics and controls engineers, astrophysicists, and the like would benefit from adding *Visualizing Quaternions* to their working library.

—Reviewed by Pierre Larochelle

REFERENCES

[1] J.B. Kuipers, *Quaternions and Rotation Sequences: A Primer with Applications to Orbits, Aerospace, and Virtual Reality*. Princeton, NJ: Princeton Univ. Press, 1999.

[2] J.M. McCarthy, *Introduction to Theoretical Kinematics*. Cambridge, MA: MIT Press, 1990.

[3] O. Bottema and B. Roth, *Theoretical Kinematics*. New York: Dover, 1990 (reprint).

[4] S. Altmann, *Rotations, Quaternions, and Double Groups*. New York: Dover, 2005 (reprint).

[5] S. Altmann, "Hamilton, rodrigues, and the quaternion scandal," *Math. Mag.*, vol. 62, no. 5, pp. 291–308, 1989.

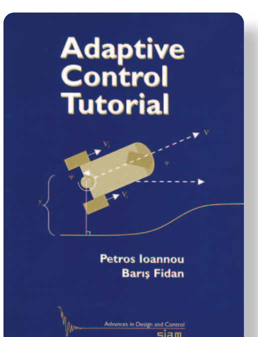
[6] J. Dai, "An historical review of the theoretical development of rigid body displacements from Rodrigues parameters to the finite twist," *Mechanism Mach. Theory*, vol. 41, pp. 41–52, 2006.

[7] P.R. Girard, *Quaternions, Clifford Algebras, and Relativistic Physics*. Basel: Birkhauser, 2007.

[8] J.A. Conway and D. Smith, *On Quaternions and Octonions*. AK Peters, 2003.

REVIEWER INFORMATION

Pierre Larochelle is a professor in the Department of Mechanical and Aerospace Engineering at the Florida Institute of Technology with expertise in robotic mechanical system design. At Florida Tech he is the founder and director of the Robotics and Spatial Systems Laboratory (RASSL). He received the Ph.D. in 1994 from the Mechanical & Aerospace Engineering Department at the University of California-Irvine. He is an associate editor for the *ASME Journal of Mechanical Design* as well as the journal *Mechanics Based Design of Structures amd Machines*. He is a member of Tau Beta Pi, Pi Tau Sigma, ASME, ASEE, and IEEE. He can be reached at pierrel@fit.edu.



SIAM, 2006
ISBN 0898716152
US$99, 389 pages.

Adaptive Control Tutorial

by PETROS IOANNOU and BARIS FIDAN

Adaptive control is a branch of modern control methodologies, with a mature theoretical foundation. Like other control methodologies, adaptive control relies on feedback of system signals. The unique feature of adaptive control is its capacity for adaptation to handle system uncertainties. An adaptive controller can guarantee the desired system stability and tracking performance in the presence of large-system parameter uncertainties, which is desirable for many performance-critical applications. Adaptive control has experienced advances and successes in both theory and applications and is developing rapidly with the emergence of new problems and solutions. Despite the vast amount of literature on both theory and applications, there is still a high demand for a comprehensive and pragmatic understanding and presentation of adaptive control theory, technical issues, and design techniques.

Adaptive Control Tutorial by Petros Ioannou and Baris Fidan is an excellent manuscript for meeting such a demand. The purpose of this book is to present the fundamental techniques and algorithms of adaptive control in a tutorial manner, with the aim of serving a wide audience, including engineers, students, and researchers who are interested in adaptive control for applications, learning, and advanced research. With eight

Digital Object Identifier 10.1109/MCS.2008.925222